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Stresses in Concrete Pipes Due to External Pressure

Municipal & Sanitary Engineering

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STRESSES IN CONCRETE PIPES DUE TO EXTERNAL PRESSURE

BY

WILLIS APPLEFORD SLATER

THESIS

FOR

DEGREE OF BACHELOR OF SCIENCE

IN

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

WILLIS APPLEFORD SLATER

ENTITLED STRESSES IN CONCRETE PIPES DUE TO EXTERNAL PRESSURE

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE

OF Bachelor of Science in Municipal and Sanitary Engineering

A N Talbot

HEAD OF DEPARTMENT OF Municipal and Sanitary Engineering

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Handling the Test-piece See p 33

Inside diam. 48"—Wall thickness 10."

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CHAPTER I.

So far as known no investigation has ever been made of the stresses in a sewer due to the exterior load upon it. However, many concrete sewers have been reinforced, the reinforcement being placed evidently without any definite knowledge of the location and magnitude of the maximum stresses. This thesis has been undertaken to determine (1) How large is the maximum stress? (2) Where does the maximum stress occur? Having determined these points it is hoped they will throw light on; whether reinforcement of sewers is advisable, if so how much reinforcing to use and where to place it.

The investigation was conducted along two lines; (1) An Analytical determination of the stresses from theoretical considerations only. (2) A series of experiments testing the correctness of the analytical determination.

I. ANALYTICAL DETERMINATION OF STRESSES.

The mathematical investigation comprises the determination of stresses due to loading in each of the following manners: (1) A vertical load concentrated upon a line in the crown of the sewer and parallel to the sewer's axis. (2) A vertical load distributed uniformly over the horizontal projection of the sewer. (3) A vertical load distributed uniformly as just described combined with a horizontal load uniformly distributed upon a vertical projection of the sewer.

The uncertainty as to stresses in sewers arises from two causes, viz., the lack of definite knowledge as to stresses developed when the condition of loading is known and the lack of knowledge of the distribution and magnitude of the load itself. Since this is true it was thought best to eliminate the second source of uncertainty and try to solve only the problem of stresses when the condition of loading is known, leaving the greater problem of distribution of earth pressure to a solution by itself. For this reason the first method of loading was used for the mathematical investigation and the two others as more nearly approximating that found in practice.

The mathematical work which follows is largely an appropriation to the case in hand of Bach's formula for moments and stresses in elastic rings, the development of the formula, $S = \frac{P}{F}\left\{1 + \frac{M}{r}\left(1 + \frac{1}{H}\frac{n}{r+n}\right)\right\}$,being taken almost verbatim from the thesis of Prof. G. A. Goodenough which was submitted in 1901 to the University of Illinois for the degree of M.E., except that such changes are made as are necessary to make it apply to compression instead of tension. From this point on the same general methods are used as are used in the above named article.

2. TESTING OF THEORY.

In testing the theory, for reasons previously explained, the loads applied were concentrated along a line of the crown of the test piece. Most of the tests were upon plain concrete because it was considered that there would be fewer complications in the stresses to obscure any agreement with the theory than

with reinforced concrete. Likewise a small number of sizes was used because better comparison of results could be made, there being fewer chances for variation in the conditions. The age of the test pieces was in the neighborhood of thirty days when broken.

CHAPTER II.

(3) NOMENCLATURE.

In fig. 1 consider BCC_1B_1 as an unstressed differential section of a unit length of elastic pipe which has a symmetrical compressive load applied.

The following nomenclature is used.

ϵ_0 1. E_0= relative compression of center line OO_1.

ϵ_f 2. E_f = relative compression of any fiber.

3. ϕ = angle at center between radius at section under consideration and the horizontal.

4. $d\phi$ = angular length of fiber OO_1.

5. $\Delta d\phi$ = increment in angular length of fiber OO_1.

6. $w = \dfrac{\Delta d\phi}{d\phi}$

7. ds = length of fiber OO_1

8. n = distance of fiber from center of gravity of section, being + when taken toward convex ~~curves~~ and - when taken toward concave side.

9. e = distance of extreme fiber from C. of G. of section.

10. r = mean radius of ring.

11. E = modulus of elasticity.

12. $H = -1/f \dfrac{n}{r+n}$

13. S = stress normal to the section considered.

14. P = pressure normal to the section considered.

15. M = moment at the section considered.

16. M_A = moment at the end of horizontal diameter.

17. Q = total load upon crown of sewer when load is concentrated.

18. p = load upon sewer per unit of area of horizontal projection when vertical load is uniformly distributed.

19. h = horizontal load upon sewer, per unit of area of vertical projection of sewer.

20. $D = 1 + \frac{1}{H}\,\frac{n}{r+n}$ 22. $K = .6362 + .6366\,\frac{H}{H+1}$

21. $F = 1 - \frac{1}{H}\,\frac{n}{r-n}$ 23. $J = .5 - .25\,\frac{H-1}{H+1}$

24. f = area of section of wall 1 inch long.

4. DEVELOPMENT OF BACH'S GENERAL EQUATION OF STRESSES.

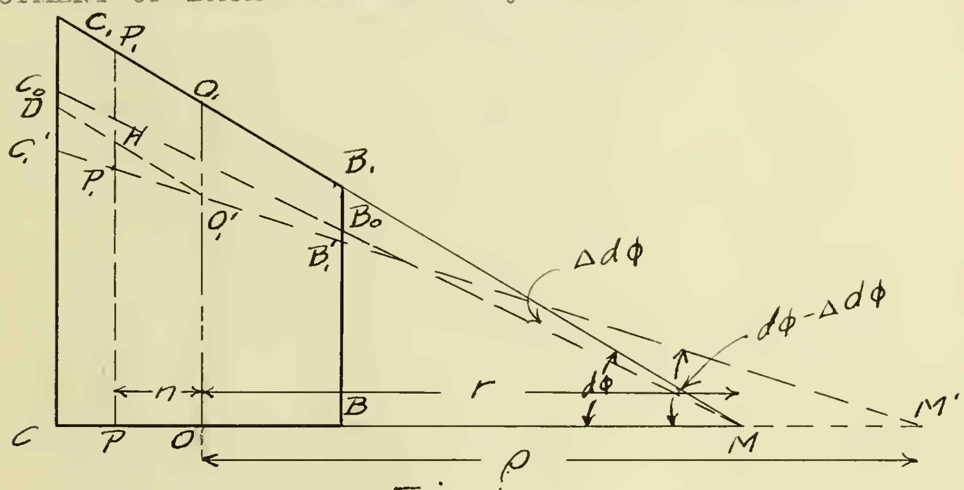


Fig. 1

Let E_o = relative compression of center line OO_1.

ds = original length of fiber OO_1.

E_f = relative compression of any fiber other than center.

$E_o = \frac{\Delta ds}{ds} = \frac{O_1O_1'}{OO_1}$ and $E_f = \frac{P_1P_1'}{PP_1}$

Draw thru O_1', $O_1'D$ parallel to C_1M.

Then $P_1P_1' = P_1H + HP_1' = O_1O_1' + HP_1'$

But $O_1O_1' = E_o\overline{OO_1} = E_o ds = E_o r d\phi$

and from the geometry of the figure

$HP_1' = O_1'H(d\phi - \{d\phi - \Delta d\phi\}) = n\,\Delta d\phi \quad \therefore P_1P_1' = E_o r\, d\phi + n\,\Delta d\phi$

and

$$PP_1' = (r+n)\,d\phi$$

Substituting these values in the expression for E_f

$$E_f = \frac{E_o\, r\, d\phi + n\,\Delta d\phi}{(r+n)\,d\phi} = \frac{E_o + \frac{n}{r}\,\frac{\Delta d\phi}{d\phi}}{1 + \frac{n}{r}}$$

Let the ratio $\frac{\Delta d\phi}{d\phi} = W$

Then $E_f = E_o + (W - E_o)\,\frac{n/r}{1 + n/r}$

The normal stress corresponding to compression E is

$$S = E\,E_f = E\left\{E_o + (W - E_o)\,\frac{n}{r+n}\right\}$$

In which E denotes the modulus of elasticity.

Placing the stresses in equilibrium with the external forces, we obtain

$$P = \int S\,df = \int E\left\{E_o + (W - E_o)\,\frac{n}{r+n}\right\} df,$$

$$M = \int n\,S\,df = \int E\,n\left\{E_o + (W - E_o)\,\frac{n}{r+n}\right\} df.$$

Assuming the modulus E to be constant.

$$P = E\left\{E_o \int df + (W - E_o)\int \frac{n}{r+n}\,df\right\}.$$

$$M = E\left\{E_o \int n\,df + (W - E_f)\int \frac{n^2}{r+n}\,df\right\}$$

The centerline OO, passes thru the center of gravity of each normal section: hence

$$\int n\,df = 0$$

Let $\int \frac{n}{r+n}\,df = -Hf.$

Then $\int \frac{n^2}{r+n}\,df = \int \left(n - r\,\frac{n}{r+n}\right) df = -r\int \frac{n}{r+n}\,df = Hfr.$

Introducing these values for the integrals in the preceding expressions for P and M

$$P = Ef\left\{E_o - H(w - E_o)\right\}$$

$$M = Ef\,(w - E_o)\,Hr.$$

Solving for E_o and w we obtain the following important equations.

$$\left.\begin{aligned} w - E_o &= \frac{M}{EfHr} \\ E_o &= \frac{P}{Ef} + H(w - E_o) = \frac{1}{Ef}\left(P + \frac{M}{r}\right) \\ w &= E_o + \frac{1}{Ef}\cdot\frac{M}{Hr} = \frac{1}{Ef}\left(P + \frac{M}{r} + \frac{M}{Hr}\right) \end{aligned}\right\} \quad 5$$

Substituting these values of E_o and w in 2

$$S = \frac{1}{f}\left(P + \frac{M}{r} + \frac{M}{Hr}\,\frac{n}{r+n}\right) \quad \text{or} \qquad \text{(A)}$$

$$S = \frac{P}{f} + \frac{M}{fr}\left(1 + \frac{1}{H}\,\frac{n}{r+n}\right)$$

This is the equation giving the total stress at any section where

f = area of section

r = radius of center of gravity of section.

H = a constant for any given pipe depending upon radius and thickness of section.

Evidently it is quite similar in form to the equation $S = \frac{P}{A} + \frac{Mc}{I}$ to which the above formula will reduce if r becomes ∞.

When r=10, e=2 and the load is 100 lbs. the results of these formulas vary about 30 % from each other.

The value of S is for any fiber between the center line OO_1 of the section and the center H. If the fiber be taken outside of the center line the equation becomes

$$S = \frac{1}{f}\left\{P + \frac{M}{r}\left(1 - \frac{1}{H}\,\frac{n}{r+n}\right)\right\}.$$

5. SOLUTION FOR H.

The sections considered will be rectangular.

Take a section of thickness $2e$ and length, unity.

Consider an elementary strip of width dn and distant n from center line.

df = dn = area of strip

$$H = -\frac{1}{f}\int_{-e}^{+e}\frac{n}{r+n}\,df = -\frac{1}{f}\int_{-e}^{+e}\frac{n}{r+n}\,dn$$

$$H = -\frac{1}{f}\Big\{r+n - r\log(r+n)\Big\}_{-e}^{+e}$$

$$= -\frac{1}{2e}\Big[\{r+e - r\log_e(r+e)\} - \{r-e - r\log(r-e)\}\Big]$$

$$= -\frac{1}{2e}\Big\{2e + r\log_e(r-e) - r\log(r+e)\Big\}$$

$$= -\frac{1}{2e}\Big\{2e + r\big[\log_e(r-e) - \log(r+e)\big]\Big\}$$

$$= -\frac{1}{2e}\Big\{2e + r\log_e\frac{r-e}{r+e}\Big\}$$

$$H = -\frac{1}{2e}\Big\{2e + \log_e\Big(\frac{r-e}{r+e}\Big)^r\Big\}$$

$$= \frac{r}{2e}\log_e\frac{r+e}{r-e} - 1 \qquad (6)$$

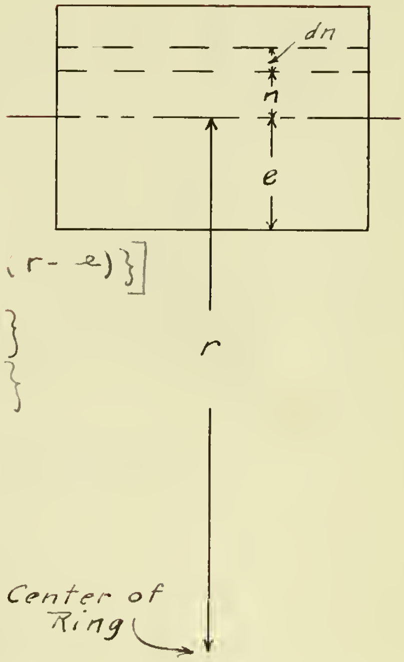


Fig. 2.

6. MOMENTS DUE TO CONCENTRATED LOAD.

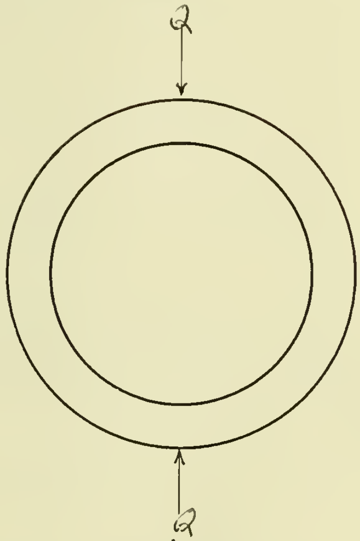


Fig. 4

Consider a ring acted upon by a concentrated load both at top and bottom as shown in Fig.3 If a section be cut from this ring as shown in Fig. (4) it will be acted upon by the forces shown at section A, a known force $\frac{Q}{2}$ and an unknown moment M_A and at the section O by a vertical force $\frac{Q}{2}$ and an unknown moment M. Now considering the section in equilibrium under these forces and taking moments about O we have

$$M_A - M + \frac{Q}{2}(r-x) = 0\,; \qquad x = r\cos\phi$$

$$M = -\frac{Qr}{2}(1-\cos\phi) - M_A \qquad (7)$$

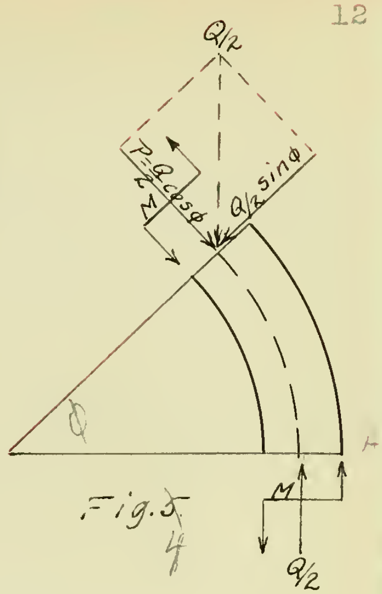


Fig. 5
4

But $\int_0^{\pi/2} \Delta \, d\phi = 0$ because the axes are always at right angles

Since $W = \frac{\Delta \, d\phi}{d\phi}$, $\int_0^{\pi/2} W \, d\phi = 0$

But

$$W \, d\phi = \frac{1}{Ef} \int_0^{\pi/2} \left\{ P + \frac{M}{r} \left(1 + \frac{1}{H}\right) \right\} d\phi = 0$$

where P = normal force at section considered.

Here $P = \frac{Q}{2} \cos\phi$

Substituting these values of P and M in the equation

$$\int_0^{\pi/2} \left[\frac{Q}{2} \cos\phi - \left\{ \frac{Qr}{2}(1 - \cos\phi) + M_A \right\} \frac{1}{r}\left(1 + \frac{1}{H}\right) \right] d\phi = 0$$

$$\frac{Q}{2} \int_0^{\pi/2} \cos\phi \, d\phi - \frac{1}{r}\left(1 + \frac{1}{H}\right) \int_0^{\pi/2} \left\{ \frac{Qr}{2}(1 - \cos\phi) + M_A \right\} d\phi = 0$$

$$\frac{Q}{2} \sin\phi \Big]_0^{\pi/2} - \left(1 + \frac{1}{H}\right) \frac{1}{r} \left[\frac{Qr}{2}(\phi - \sin\phi) + M_A \phi \right]_0^{\pi/2} = 0$$

$$\frac{Q}{2} - \frac{1}{r}\left(1 + \frac{1}{H}\right) \left[\frac{Qr}{2}\left(\pi/2 - 1\right) + M_A \frac{\pi}{2} \right] = 0$$

$$M_A \frac{\pi}{2}\left(1 + \frac{1}{H}\right) \frac{1}{r} = \frac{Q}{2} - \left(1 + \frac{1}{H}\right) \frac{Q}{2}\left(\frac{\pi}{2} - 1\right)$$

$$M_A = \frac{Qr}{\pi} \left\{ \frac{1}{1 + \frac{1}{H}} - \left(\frac{\pi}{2} - 1\right) \right\} = .3183 \, Qr \left(\frac{H}{H+1} - .5708 \right) \qquad \text{G}$$

$$M = -M_A - \frac{Qr}{2}(1 - \cos\phi) \qquad \text{See Eq. (1.)}$$

Substituting in this equation the above value of M_A

$$M = .3183 \, Qr \left(.5708 - \frac{H}{H+1} \right) - \frac{Qr}{2}(1 - \cos\phi)$$

$$= \frac{Qr}{2} \left\{ \cos\phi - .6362 - .6366 \frac{H}{H+1} \right\} \qquad \text{H}$$

$$= \frac{Qr}{2} \left\{ \cos\phi - \frac{2}{\pi}\left(1 + \frac{H}{H+1}\right) \right\}$$

7. STRESS DUE TO CONCENTRATED LOAD.

Now $S = \frac{P}{f} + \frac{M}{fr}\left(1 + \frac{1}{H} \frac{\eta}{r+\eta}\right)$

$P = \frac{Q}{2} \cos\phi$

$$\therefore S = \frac{Q \cos\phi}{2f} + \frac{Qr}{2fr} \left\{ \cos\phi - \frac{2}{\pi}\left(1 + \frac{H}{H+1}\right) \right\} \left(1 + \frac{1}{H} \frac{\eta}{r+\eta}\right)$$

$$S = \frac{Q}{2f}\left(\cos\phi + \left\{\cos\phi - \frac{2}{\pi}\left(1 + \frac{H}{H+1}\right)\right\}\left(1 + \frac{1}{H}\,\frac{n}{r+n}\right)\right)$$ J

This is the equation that will be used for obtaining ~~st~~ stresses when n is positive, i.e., outside the center line of the section.

When n is negative, i.e., between the centerline of the section and the center of the circle, the equation becomes

$$S = \frac{Q}{2f}\left\{\cos\phi + \left(\cos\phi - \frac{2}{\pi}\left(1 + \frac{H}{H+1}\right)\right)\left(1 - \frac{1}{H}\,\frac{n}{r-n}\right)\right\}$$

When S is positive the stress is tension.

When S is negative the stress is compression.

If in the foregoing equations we take

(1) $1 + \frac{1}{H}\,\frac{n}{r+n} = D$

(2) $1 - \frac{1}{H}\,\frac{n}{r-n} = F.$

(3) $\frac{2}{\pi}\left(1 + \frac{H}{H+1}\right) = .6364\left(1 + \frac{H}{H+1}\right) = K$

we have them reduced to the following forms.

(1) $S = \frac{Q}{2f}\left\{\cos\phi + D(\cos\phi - K)\right\}$ for outer fiber. k

(2) $S = \frac{Q}{2f}\left\{\cos\phi + F(\cos\phi - K)\right\}$ for inner fiber. k'

8. MOMENTS DUE TO UNIFORM LOAD.

(a) By Bach's Method.

Consider a sewer acted upon as shown in Fig. 5 by a uniform load, for the purpose of simplicity considering no horizontal thrust as acting.

If p = the uniform load, the load borne at sections A and B will each be pr.

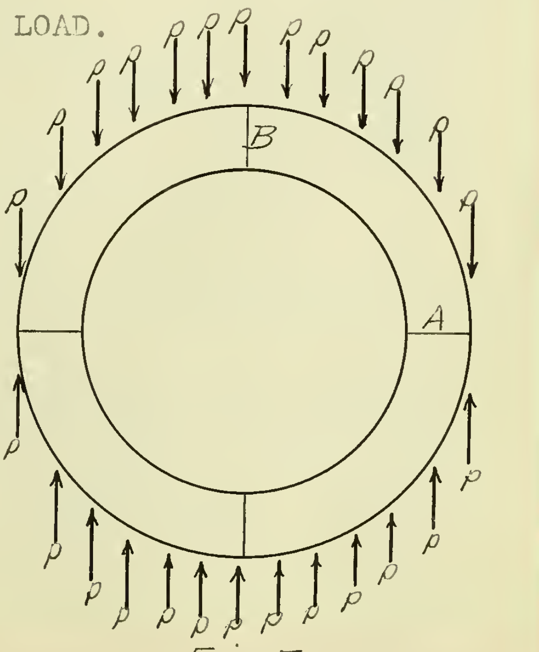


Fig.5

If a section be cut from a sewer as shown in Fig. 6 it will be acted upon by a known force pr and an unknown moment M_A at section A of Fig. 4; at section O, a force px and a moment M and on the perimeter by the uniform load p(r-x). Now considering the section in equilibrium under these forces, and taking moments about O, we have $M + M_A + pr(r-x) - p\frac{(r-x)^2}{2} = 0$

The force pr may be resolved into its components respectively normal and parallel to section O and equal to px $\cos\phi$ and px $\sin\phi$ producing normal and shearing stresses. The shear px $\sin\phi$ will be neglected.

$$M + M_A + pr(r-x) - p\frac{(r-x)^2}{2} = 0$$

$$M = p\frac{(r-x)^2}{2} - pr(r-x) - M_A.$$

But $\int_0^{\pi/2} \Delta\, d\phi = 0$ because the major axes are always at right angles.

Since $w = \frac{\Delta\, d\phi}{d\phi}$, $\int_0^{\pi/2} w\, d\phi = 0$

From the 3rd of equations 5

$$w\, d\phi = \frac{1}{Ef}\int_0^{\pi/2}\left\{P + \frac{M}{r}\left(1 + \frac{1}{H}\right)\right\} d\phi = 0$$

But in this case $P = px\cos\phi$

Substituting these values of P and M

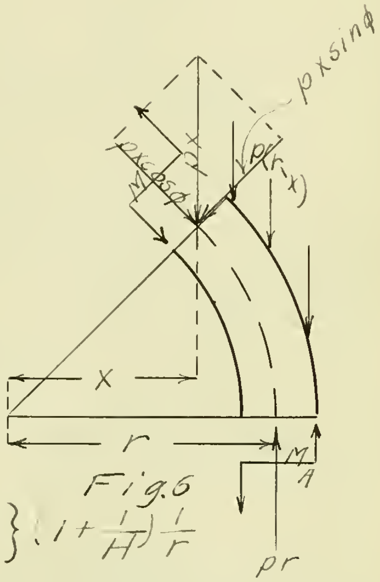


Fig.6

$$w\, d\phi = \frac{1}{Ef}\int_0^{\pi/2}\left\{px\cos\phi + \left\{p\frac{(r-x)^2}{2} - pr(r-x) - M_A\right\}\left(1+\frac{1}{H}\right)\frac{1}{r}\right.$$

But $x = r\cos\phi$

$$w\, d\phi = \frac{1}{Ef}\int_0^{\pi/2}\left\{pr\cos^2\phi + \left[pr\frac{(1-\cos\phi)^2}{2} - pr^2(1-\cos\phi) - M_A\right]\left(1+\frac{1}{H}\right)\frac{1}{r}\right\} d\phi = 0$$

$$pr\int_0^{\pi/2}\cos^2\phi\, d\phi + \left[pr^2\int_0^{\pi/2}\left\{\frac{(1-\cos\phi)^2}{2} - 1 + \cos\phi\right\} - \int_0^{\pi/2} M_A\right]\left(1+\frac{1}{H}\right)\frac{1}{r}\, d\phi = 0$$

$$pr\left(\frac{\phi}{2} + \frac{1}{4}\sin 2\phi\right)_0^{\pi/2} + pr\left(1+\frac{1}{H}\right)\left(\frac{\phi}{2} - \sin\phi + \frac{\phi}{4} + \frac{1}{8}\sin 2\phi\right.$$

$$\left. - \phi + \sin\phi\right)_0^{\pi/2} = \left(\frac{M_A}{r}\right)_0^{\pi/2}\left(1+\frac{1}{H}\right)_0^{\pi/2}$$

$$\frac{M_A}{r}\,\frac{\pi}{2}\left(1+\frac{1}{H}\right)=pr\left(\frac{\pi}{4}+0\right)+pr\left(1+\frac{1}{H}\right)\left(\frac{\pi}{4}-1+\frac{\pi}{8}+0-\frac{\pi}{2}+1\right)$$

$$\frac{M_A}{r}=\frac{pr\left(\pi/4+\left[1+\frac{1}{H}\right)\left(\pi/4+\pi/8-\pi/2\right)\right.}{\pi/2\left(1+\frac{1}{H}\right)}=\frac{2pr\left(1/4+\left[1/4+1/8-1/2\right]\left[1+\frac{1}{H}\right]\right)}{1+\frac{1}{H}}$$

$$M_A=\frac{.5pr^2}{1+\frac{1}{H}}-.25pr^2$$

$$=+.25pr^2\frac{H-1}{H+1} \qquad \text{(B)}$$

From this equation the value of the moment at any point may be obtained by substituting this value of M_A in the equation

$$M=p\frac{(r-x)^2}{2}-pr(r-x)-M_A \quad \text{where } x=r\cos\phi.$$

$$\therefore M=pr^2\frac{(1-\cos\phi)^2}{2}-pr^2(1-\cos\phi)-pr^2\left(\frac{.5H}{H+1}-.25\right)$$

$$=.5pr^2(\cos^2\phi-1-.25pr^2\frac{H-1}{H+1})$$

$$=-pr^2\left(.5\sin^2\phi+.25\frac{H-1}{H+1}\right) \qquad \text{(C)}$$

$\frac{H-1}{H+1}$ is always a negative quantity so M is always less than $.5pr^2$

Calling the moment for $\phi=90°$, M_B we have

$$M_B=-pr\left(0+.25\frac{H-1}{H+1}\right)=-.25pr^2\frac{H-1}{H+1}$$

(b) MOMENTS BY CORNELL METHOD.

The correctness of these values of M and M_A can be checked by another method.

If M = the moment at any section of a stressed ring, ds = the linear deformation of a differential portion of the ring as in the previous demonstration and $d\phi$ = the angular deformation, then $\frac{M\,ds}{EI}=d\phi$ (See Church's Mechanics of EngineeringI pp 445-449 or Art. by Filkins & Fort in Transactions of the Association of Civil Engineers of Cornell University 1895-'96 pp. 99-101.)

~~Assume the following equations as proved in the Cornell article for any condition of loading.~~

Then consider Fig. 1, Plate VIII as a free body under a uniform load of p per unit horizontal distance with uniformly distributed reaction. Then the body will be in equilibrium under the load pr, its reaction pr, a horizontal ~~stress~~ pressure T_B and T_D and unknown moments M_B and M_D as shown in the figure.

From the symmetry of the figure $T_B = T_D$

But these being the only horiz. forces the body is in equilibrium under them.

$$\therefore T_B + T_D = 0 \text{ and } T_B = T_D = 0$$

Also for conditions of equilibrium.

$$M_B = M_D$$

Likewise in fig. 2 for equilibrium

$$J_A = T_B = 0$$

$$T_A = pr$$

Clockwise moments will be considered positive and counter clockwise moments negative in this discussion. Also all couples assumed as acting at sections shown in figures will be assumed as positive and when the expression for each is computed, the sense of the moment will be taken care of by the sign.

Then in Fig. 2 taking moments about B,

$$M_A + M_B + \Sigma p \frac{r}{2} - T_A r = 0$$

$$M_B + M_A + p \frac{r^2}{2} - pr^2 = 0$$

$$M_A = pr^2 - \frac{pr^2}{2} - M_B = \frac{pr^2}{2} - M_B$$

The following values have now been deduced.

$$T_B = 0; \quad T_D = 0$$

$$J_A = 0$$

$$T_A = pr$$

$$M_A = \frac{pr^2}{2} - M_B$$

$$M_B = \frac{pr^2}{2} - M_A$$

DETERMINATION OF M_A and M_B.

In Fig. 4 let O be any section and M be the moment in the section. Consider the origin of the co-ordinate axes at the center of the circle of which this is an arc. Taking moments about O

$$M + M_B - \frac{px^2}{2} = 0$$

$$M = \frac{px^2}{2} - M_B.$$

From Eq. I, $\frac{M\,ds}{EI} = 0, \quad \therefore \int M\,ds = 0$

$$ds = \sqrt{dx^2 + dy^2}$$

$$dy = -\frac{x}{y}\,dx; \quad dy = \frac{x}{\sqrt{r^2 - x^2}}\,dx.$$

$$ds = \sqrt{dx^2 + \frac{x^2dx^2}{r^2 - x^2}} = \frac{r\,dx}{\sqrt{r^2 - x^2}}$$

Substituting these values for M and ds. in Eq. I

$$\int_0^r \left(\frac{px^2}{2} - M_B\right)\frac{r\,dx}{\sqrt{r^2 - x^2}}\,dx = 0$$

$$M_B r \int_0^r \frac{dx}{\sqrt{r^2 - x^2}} = \frac{pr}{2}\int_0^r \frac{x^2\,dx}{\sqrt{r^2 - x^2}}$$

$$M_B r\left(\sin^{-1}\frac{x}{r}\right)_0^r = \frac{pr}{2}\left(-\frac{x}{r}\sqrt{r^2 - x^2} + \frac{r^2}{2}\sin^{-1}\frac{x}{r}\right)_0^r$$

$$M_B r\pi/2 = \frac{pr}{2}\left(\frac{r^2}{2}\pi/2\right) = \frac{pr^3\pi}{8}$$

$$M_B = .25\,pr^2$$

But $M_A = .5pr^2 - M_B = .5pr^2 - .25pr^2 = .25pr.^2$

This does not seem to be as it ought to be because there is evi-

dently a change in sign between A and B but this analysis does not show it. Likewise in the Cornell article by Filkins and Fort, (See Transactions of Cornell Association of Civil Engineers 1895-96 pp 102 and 3) the same thing is done for a concentrated load at the crown. The moment at the top is determined and from that the moment at the side is found and is of the same sign; yet it is considered and shown by them later that there is a point of zero moment between these sections. In the present problem if M_A is solved for directly as M_B was found, it is found to be $-.25\ pr^2$ as would be expected but why the two methods do not correspond is the question. The direct method of obtaining M_B is herewith given.

In Fig. 3 let the origin be at A and let M be the moment at any section O.

Then $M + M_A + prx - \frac{px^2}{2} = 0$

$$M = \frac{px^2}{2} - prx - M_A$$

But $\int M\,ds = 0$

$$ds = \sqrt{dx^2 + dy^2}$$

$$dy = \frac{(r-x)\,dx}{\sqrt{2rx - x^2}}\ , \quad x^2 + y^2 = 2rx \text{ being the equation of the circle}$$

$$\therefore ds = \frac{r\,dx}{\sqrt{2rx - x^2}}$$

Substituting these values of M and ds

$$pr\int_0^r \frac{x^2\,dx}{\sqrt{2rx-x^2}} - pr^2\int_0^r \frac{x\,dx}{\sqrt{2rx-x^2}} - M_A\,r\int_0^r \frac{dx}{\sqrt{2rx-x^2}} = 0$$

Integrating this we obtain

$M_A = -.25pr^2$ as would be expected.

These same values are given for M_A and M_B in Turneaure and Russell's work on Water Supply p. 499 but the derivation is not given.

DETERMINATION OF MOMENT AT ANY POINT.

The general expression for moment at any point will now be derived. In Fig. 4 let the origin be at the center of the circle and let M be the moment at any section O. Taking moments about O

$$M + M_B - \frac{\rho x^2}{2} = 0$$
$$M = \frac{\rho x^2}{2} - M_B$$
$$x = r \sin \phi$$
$$M_B = .25 \rho r^2$$
$$\therefore M = \rho r^2 \left(\frac{\sin^2 \phi}{2} - .25 \right)$$

To find the section of zero moment put M = o

Then $\frac{\sin^2 \phi}{2} - .25 = 0$

$$\sin^2 \phi = .5$$
$$\phi = 45° 0'$$

SUMMARY OF VALUES.

The following values of Moments have been obtained.

$$M_B = .25 \rho r^2$$
$$M_A = -.25 \rho r^2$$
$$M = \rho r^2 \left(\frac{\sin^2 \phi}{2} - .25 \right)$$

Section of zero moment is $\phi = 45°$.

(c) COMPARISON OF RESULTS FROM THESE METHODS.

A comparison of these values with those obtained by the previous method shows a close correspondence.

$$\text{I} \quad M_A \begin{cases} = .25\, pr^2 \dfrac{H-1}{H+1} & \text{Back's method} \\ = .25\, pr^2 & \text{Cornell method.} \end{cases}$$

$$\text{II} \quad M \begin{cases} = -pr^2(.5 \sin^2\phi + .25\ \dfrac{H-1}{H+1}) \\ = -pr^2(.5 \sin^2\phi - .25) \end{cases}$$

These two methods were worked out independently and the nomenclature was not the same so the difference that appears is a difference in nomenclature. When reduced to the same terms the above correspondence is found. It is shown later that H has a value (for probable values of e/r) ranging between .0025 and .005. For the largest of these values $\left(\frac{H-1}{H+1} = \frac{.995}{1.005}\right)$ =.99, which it is $\frac{1-H}{1+H}$ seen brings these two values together very closely.

We now proceed to obtain from the first set of values derived for M and M_A the equation representing the stress at any section.

9. STRESSES DUE TO UNIFORM LOAD.

We have from Eq. A p.10

$$S = \frac{P}{f} + \frac{M}{fr}\left(1 \pm \frac{1}{H}\,\frac{n}{r+n}\right)$$

and from equation C p.15

$$M = -pr^2(.5 \sin^2\phi + .25\ \frac{H-1}{H+1})$$

Substituting this value of M in the equation for S, we obtain

$$S = \frac{P}{f} - \frac{pr}{f}(.5 \sin^2\phi + .25\,\frac{H-1}{H+1})(1 \pm \frac{1}{H}\,\frac{n}{r+n}) \qquad \text{D}$$

But $P = 2pr\cos^2\phi$

Substituting these values in equation D

$$S = \frac{\rho r \cos^2\phi}{f} - \frac{\rho r}{f}\left(\frac{1-\cos^2\phi}{2} + .25\frac{H-1}{H+1}\right)\left(1+\frac{1}{H}\frac{n}{r+n}\right)$$

$$= \frac{\rho r}{f}\left\{\cos^2\phi + \left(.5\cos^2\phi - \left[.5+.25\frac{H-1}{H+1}\right]\right)\left(1+\frac{1}{H}\frac{n}{r+n}\right)\right. \qquad E$$

Eq. E is the general equation of the stress in any fiber. If the quantity n has a negative instead of a positive value as in equation E the form becomes that of E'.

$$S = \frac{\rho r}{f}\left\{\cos^2\phi + \left(.5\cos^2\phi - \left[.5+.25\frac{H-1}{H+1}\right]\right)\left(1-\frac{1}{H}\frac{n}{r-n}\right)\right\} \qquad E'$$

n is considered positive when the fiber considered is between the center of gravity of the section and the outside circumference, and negative if it lies between the centerline of the section and the inner circumference.

~~This distinction is shown plainly in fig. 7~~

If in equations E and E' we make the following substitutions where n=e

$$.5 + .25\frac{H-1}{H+1} = J$$

$$1+\frac{1}{H}\frac{n}{r+n} = D$$

$$1-\frac{1}{H}\frac{n}{r-n} = F$$

we have the following equations.

$$S = \frac{\rho r}{f}\left\{\cos^2\phi + (\tfrac{1}{2}\cos^2\phi - J)D\right\} \qquad \text{Outer fiber} \quad F$$

$$S = \frac{\rho r}{f}\left\{\cos^2\phi \pm (\cos^2\phi - J)F\right\} \qquad \text{Inner fiber} \quad F'$$

10. MOMENTS AND STRESSES DUE TO COMBINED VERTICAL AND HORIZONTAL LOAD.

It is desired to obtain a value for the moment in any section of a sewer when there is not only a vertical but also a horizontal pressure.

When this is the case we will have the condition shown in the accompanying figure. Assume that the section shown in fig. 8 is acted upon by the forces shown, p being the uniform vertical load per unit of horizontal distance and h the uniform horizontal load per unit of vertical distance. The section will be in

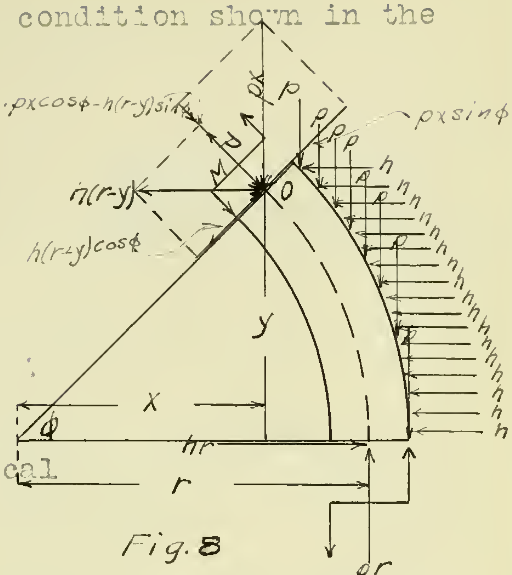


Fig. 8

equilibrium under the action of the forces pr acting normal to its section and px and h(r-y) acting at the section O which represents any section between $\phi = 0$ and $\phi = \pi/2$. But the exact points of application of pr and P are not known. They will therefore be supplied by pr and P acting at the middle of their respective sections and the couples M_A and M. Taking moments about O,

$$M_A + M + pr(r-x) - \frac{p(r-x)^2}{2} - \frac{hy^2}{2} + hry = 0$$

$$M = \frac{p(r-x)^2}{2} - pr(r-x) + \frac{hy^2}{2} - hry - M_A.$$

$$x = r\cos\phi \text{ and } y = r\sin\phi$$

$$\therefore M = pr^2\frac{(1-\cos\phi)^2}{2} - pr^2(1-\cos\phi) + \frac{hr^2\sin^2\phi}{2} - hr^2\sin\phi - M_A.$$

$$= pr^2(.5 - \cos\phi + .5\cos^2\phi - 1 + \cos\phi) + hr^2(.5\sin^2\phi - \sin\phi) - M_A.$$

$$= pr^2(.5\cos^2\phi - .5) + hr^2(.5\sin^2\phi - \sin\phi) - M_A$$

$$= \frac{pr^2}{2}(\cos^2\phi - 1) + hr^2(.5\sin^2\phi - \sin\phi) - M_A$$

It has been shown in Eq. A that

$$\int_0^{\pi/2}\left\{P + \frac{M}{r}\left(1+\frac{1}{H}\right)\right\}d\phi = 0,$$ where

$$P = px\cos\phi - h(r-y)\sin\phi,$$

$$x = r\cos\phi \text{ and } y = r\sin\phi$$

Then $P = pr\cos^2\phi - hr(1-\sin\phi)\sin\phi$
$= pr\cos^2\phi - hr(\sin\phi - \sin^2\phi)$

Substituting these values in eq. (A)

$$\int_0^{\pi/2}\left\{pr\cos^2\phi - hr(\sin\phi-\sin^2\phi) + \frac{\frac{pr^2}{2}(\cos^2\phi-1)+hr^2(.5\sin^2\phi-\sin\phi)-M_A}{r}\left(1+\frac{1}{H}\right)\right\}=0$$

$$\int_0^{\pi/2} pr\left\{\cos^2\phi+\frac12(\cos^2\phi-1)\left(1+\frac1H\right)\right\}d\phi - \int_0^{\pi/2} hr\left\{\sin\phi-\sin^2\phi - (.5\sin^2\phi-\sin\phi)\left(1+\frac1H\right)\right\} - \int_0^{\pi/2}\frac{M_A}{r}\left(1+\frac1H\right)d\phi=0$$

$$\frac1r M_A(1+1/H)\int_0^{\pi/2}d\phi = pr\int\cos\phi\, d\phi - .5pr\left(1+\frac1H\right)\int_0^{\pi/2}\sin^2\phi\, d\phi - hr\int\sin\phi\, d\phi + hr\int_0^{\pi/2}\sin^2\phi\, d\phi$$
$$+\left(1+\frac1H\right).5hr\int_0^{\pi/2}\sin^2\phi\, d\phi - \left(1+\frac1H\right)hr\int_0^{\pi/2}\sin\phi\, d\phi$$

Integrating the above equation.

(1.) $\frac1r M_A\left(1+\frac1H\right)\pi/2 = pr\left[\frac\phi2+\frac14\sin2\phi\right]_0^{\pi/2} - .5pr\left[\left(1+\frac1H\right)\left(\frac\phi2-\frac14\sin2\phi\right)\right]_0^{\pi/2} + hr\left[\frac\phi2-\frac14\sin2\phi\right]_0^{\pi/2}$
$+ hr[\cos\phi]_0^{\pi/2} + \left(1+\frac1H\right).5hr\left[\phi/2-1/4\sin2\phi + \left(1+\frac1H\right)hr[\cos\phi]_0^{\pi/2}\right.$

(2.) $\frac1r M_A\left(1+\frac1H\right)\pi/2 = pr\frac\pi4 - .5pr\left(1+\frac1H\right)\pi/4 + hr\frac\pi4 - hr + \left(1+\frac1H\right).5hr\frac\pi4 - \left(1+\frac1H\right)hr$

(3.) $\frac1r M_A\left(1+\frac1H\right)\pi/2 = pr\left\{\pi/4 - .5\left(1+\frac1H\right)\pi/4\right\} + hr\left\{\pi/4 - 1 - .5\left(1+\frac1H\right)\pi/4 - \left(1+\frac1H\right)\right\}$

(4.) $\frac1r M_A\left(1+\frac1H\right)\pi/2 = pr\frac\pi4\left\{1-.5\left(1+\frac1H\right)\right\} + hr\left\{\frac\pi4 - 1 - \left(1+\frac1H\right)\left(.5\frac\pi4+1\right)\right\}$

(5) $$M_A = \frac{pr\,\pi/4\left\{1-.5\left(1+\frac1H\right)\right\} + hr\left\{\pi/4 - 1-\left(1+\frac1H\right)\left(.5\frac\pi4+1\right)\right\}}{\left(1+\frac1H\right)\frac{\pi}{2r}}$$

(6) $$M_A = .5pr^2\frac{1}{1+\frac1H} - .25pr^2 + hr^2\left\{\frac{1}{2\left(1+\frac1H\right)} - \frac{2}{\left(1+\frac1H\right)\pi}\right\} - hr^2\left(\frac14+\frac2\pi\right)$$

(7) $$M_A = .25pr^2\frac{H-1}{H+1} + hr^2\frac{\pi(H-1)-8(2H-1)}{H+1}$$

Now if we assume that h=p it is the same as assuming an equal pressure in all directions upon the walls, but in this case the moment is 0 in all sections.

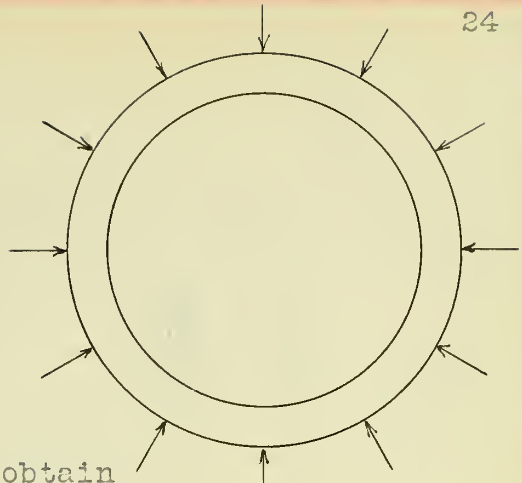


Fig.2

Therefore substituting h=p in the above equation ought to give, $M_A = 0$ regardless of the value of H since H depends only on the value of e/r.

Making this substitution we obtain

$$M_A = pr^2\left\{\left(\frac{1}{1+\frac{1}{H}}\right)\left(1-\frac{2}{\pi}\right) - .5 - \frac{2}{\pi}\right\} = 0$$

$$\therefore \left(\frac{1}{1+\frac{1}{H}}\right)\left(1-\frac{2}{\pi}\right) - .5 - \frac{2}{\pi} = 0$$

$$\frac{1}{1+\frac{1}{H}} = \frac{.5-\frac{2}{\pi}}{1-\frac{2}{\pi}} \text{ or } \frac{H}{H+1} = \frac{.5-\frac{2}{\pi}}{1-\frac{2}{\pi}}$$

$$H = \left(\frac{.5-\frac{2}{\pi}}{1-\frac{2}{\pi}}\right)(H+1) \text{ whence } H - \left(\frac{.5-\frac{2}{\pi}}{1-\frac{2}{\pi}}\right)H = 1 \text{ and}$$

$$H = 1 \div \left(1 - \frac{.5-\frac{2}{\pi}}{1-\frac{2}{\pi}}\right)$$

but this is a constant and since H cannot be a constant the value obtained for M_A is in error.

The correct value for M_A may be obtained from the analogy between the horizontal and vertical pressures. As has been shown, for uniform vertical pressure:

$$M_A = .25pr^2\frac{H-1}{H+1} \text{ and}$$

$$M_B = -.25pr^2\frac{H-1}{H+1}.$$

If the loads be applied both horizontally and vertically in a uniform manner, the end of the horizontal diameter is in the same condition under the horizontal load as is the crown under the vertical load so if h=p the total moment M'_A is the algebraic sum of the moments M_A and M_B or,

$$M'_A = .25pr^2\frac{H-1}{H+1} - .25pr^2\frac{H-1}{H+1}.$$

and for the general case of h

$$M_A = .25pr^2\frac{H-1}{H+1} - .25hr^2\frac{H-1}{H+1}; \text{ finally}$$

$$M_A = .25(p-h)r^2\frac{H-1}{H+1}. \qquad L.$$

This value of M_A may be substituted in the general equation for M due to both vertical and horizontal uniform loads,

$$M = \frac{pr^2}{2}(\cos^2\phi - 1) + hr^2(.5\sin^2\phi - \sin\phi) - M_A.$$

This value of M so obtained may in turn be substituted in the general equation of the stress in any fiber,

$$S = \frac{P}{f} + \frac{M}{fr}\left(1 + \frac{1}{H}\frac{n}{r+n}\right)$$

and a value obtained for the stress in any fiber under both a horizontal and a vertical load.

DISCUSSION OF FORMULA FOR STRESS

DUE TO VERTICAL AND HORIZONTAL LOADS.

The equation of the stress due to a vertical and a horizontal load, which would be obtained by the method just outlined, is based on the supposition that the horizontal and vertical pressures are both uniformly distributed. Probably neither one of these suppositions is true, but in the case of sewers placed at considerable depth in the ground it is probably approximately true.

The arch action of the earth upon the sewer would tend to distribute the load over the sewer in a uniform manner, and experiments seem to show that for depths below 9 or 10 ft. the horizontal pressure is approximately a constant ratio times the weight of the superimposed earth.

A series of experiments performed by Frank A Barbour, member of the Boston Society of Engineers, on the pressure of earth in trenches and reported in the Journal of the Association

of Engineering Societies for December 1897 brings out the conclusion that below seven ft. the average horizontal pressure is about .39 of the weight of superimposed earth and that for greater depths it does not vary greatly from this. If this be true, a certain ratio of h/p may be used for any particular kind of soil and it seems that a pretty fair approximation ought to be made of maximum stresses in sewers, though of course it is realized that this must involve much preliminary work with earth pressures as well as with tests of sewer under known loading.

11. DISCUSSION OF GENERAL EQUATION FOR STRESSES.

$$S = \frac{Q}{2f}\left\{\cos\phi + \left(\cos\phi - .6362 - .6366\frac{H}{H+1}\right)\left(1+\frac{1}{H}\frac{n}{r+n}\right)\right\} = \frac{Q}{2f}\left\{\cos\phi + \left[\cos\phi - \frac{2}{\pi}\left(1+\frac{H}{H+1}\right)\right]\left[1+\frac{1}{H}\frac{n}{r+n}\right]\right\}$$

The above is the general form of equation of the stress in any fiber of a sewer. Now if n be taken equal to e this equation represents the stresses in the extreme fibers. H varies only as e/r varies, and $\frac{e}{r+e}$ varies only with e/r, therefore the term $1+\frac{1}{H}\frac{e}{r+e}$ varies only with e/r and is a constant for all sewers of whatever size having the same ratio e/r. The same must necessarily be true of .6366 $\frac{H}{H+1}$ since H varies only with e/r, therefore the term $\cos\phi - .6362 - .6366\frac{H}{H+1}$ varies only $\cos\phi$ and e/r, and not at all with the size of the sewer.

Hence the whole expression

$$\cos\phi + \left(\cos\phi - .6362 - .6366\frac{H}{H+1}\right)\left(1+\frac{1}{H}\right)\frac{n}{r+n}$$

varies only with e/r and $\cos\phi$ and not at all with the size of the sewer.

12. COMPARISON OF MOMENTS IN A RING WITH MOMENTS IN A BEAM.

In all of the moment formulas for moments in a ring a quantity $\frac{H-1}{H+1}$ or $\frac{H}{H+1}$ is involved. For all probable cases H is so small that either of these quantities is approximately unity hence for simplicity these quantities will be dropped out of consideration in this comparison with beam moments. The moments in beams are given in terms of W and l where W equals the total load and l = the span. Changing the moment equations for rings to the same terms we have r= l/2 or 2r=l and for concentrated loads Q=W while for uniform load 2pr=W.

CONCENTRATED LOADS.

Moments in a ring at the crown

$$M_3 = .3182\,Qr = .3182\,W\frac{l}{2} = .16\,Wl = \frac{1}{6}\,Wl \text{ (about)}$$

Moments in a simple beam at middle

$$M = \tfrac{1}{4}\,Pl = \frac{1}{4}\,Wl$$

Max. moments in a beam fixed at both ends

$$M = \frac{1}{8}\,Pl = \frac{1}{8}\,Wl$$

A comparison of these results show that the maximum moment in a ring is a little less (1/48 Wl) than the mean of the max. moments in a simple and a restrained beam.

UNIFORM LOAD.

Moments in a ring at the crown

$$M = .25\,pr^2 = \frac{W}{8} \times \frac{l}{2} = \frac{1}{16}\,Wl$$

Moments in a simple beam at middle

$$M = \frac{1}{8} Wl$$

Max. Moments in a beam fixed at both ends.

$$M = \frac{1}{12} Wl$$

13. SUMMARY OF RESULTS.

For convenience of reference, the following summary is given, of the results used in the determination of stresses.

(6); $H = \frac{r}{2e} \log\left(\frac{r+e}{r-e}\right) - 1$ (p.11)

(B); $M_A = .25pr^2 \frac{H-1}{H+1}$ (p. 16) for uniform vertical load.

(G.); $M_A = .3183\left(\frac{H}{H+1} - .5708\right) Qr$ (p.12) " concentrated " ".

(L); $M_A = .25(p-h)r^2 \frac{H-1}{H+1}$; for both vertical and horizontal load.

(C.); $M = -pr^2(.5\sin^2\phi + .25\frac{H-1}{H+1})$ (p.5); uniform load.

(H); $M = \frac{Qr}{2}\left\{\cos\phi - \frac{2}{\pi}\left(1+\frac{H}{H+1}\right)\right\}$ (p.2) concentrated "

(F); $S = \frac{pr}{f}\left\{\cos^2\phi + (\cos^2\phi - J)D\right\}$ (p.21) outer " uniform load.

(F'); $S = \frac{pr}{f}\left\{\cos^2\phi + (\cos^2\phi - J)F\right\}$ (p.21) inner " "

(K) $S = \frac{Q}{2f}\left\{\cos\phi + D(\cos\phi - K)\right\}$ (p.13) outer fiber concentrated load.

(K') $S = \frac{Q}{2f}\left\{\cos\phi + F(\cos\phi - K)\right\}$ (p.13) inner " "

CHAPTER III.

14. DESCRIPTION OF THE TEST PIECES.

The theory of the strength of sewer pipe which has been deduced was tested by the breaking of ten pieces of concrete sewer pipe. A much larger number of tests would have been preferable but lack of time for making and breaking the pieces prevented this. Of these ten pieces, eight were made by the writer, of plain concrete in the Laboratory of Applied Mechanics of the University of Illinois and two were reinforced concrete pipes made by the "Reinforced Concrete Pipe Co." of Jackson, Michigan.

The principal tests were on plain concrete because, as has been before stated, the main object of these investigations has been to determine the stresses under known conditions of loading and it was believed this would be much less complicated in the case of plain than of reinforced concrete.

The following table gives the sizes and number of test pieces.

No. of pieces	Inside diam. inches	Wall Thickness Inches	By Whom Made
2	15	$1\frac{7}{8}$	Jackson S.P.Co.
2	24	3	U. of I.
2	24	5	U. of I.
2	48	6	U. of I.
2	48	10	U. of I

It is seen that only two inside diameters are used and two ratios of wall thickness to diameter. This small number of sizes was used because it was believed that a better comparison

of results could be made when there were few varying conditions to obscure any variation from the theory.

Because of a lack of time the age of the plain concrete test pieces was limited to thirty days.

15. MATERIALS USED.

A Portland cement was used locally known as Joint Committee cement. It is a mixture of several different kinds of cement.

The sand was a gravelly sand sifted through a quarter inch sieve and containing 28% of voids. Below is a table showing the analysis for fineness.

No. of sieve	Percent passing.
4	100
10	73
20	36
50	12
74	5
100	2

The stone was broken Kankakee limestone passing a 1 inch sieve and caught on a quarter inch sieve and containing 47% of voids.

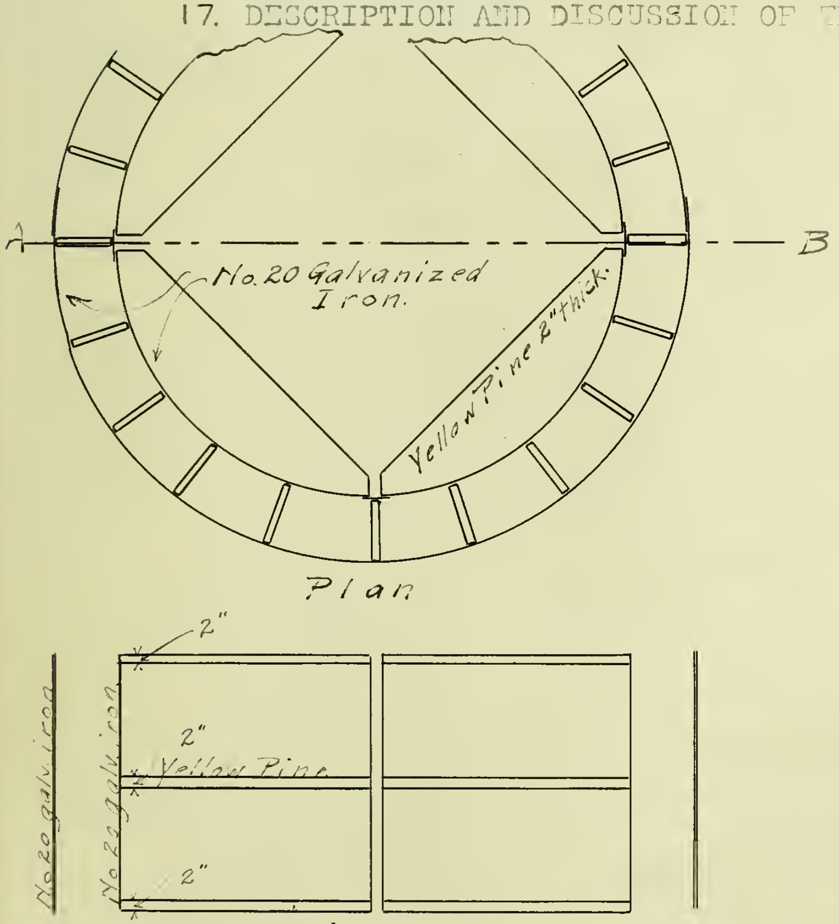
The proportions used were 1,- 2,- 4 and enough water was added to flush to the surface on tamping.

All concrete was spaded before tamping, tamped, spaded again and tamped again. In general the test pieces looked very well when the forms came off but in a few cases they showed a lack of tamping near the bottom.

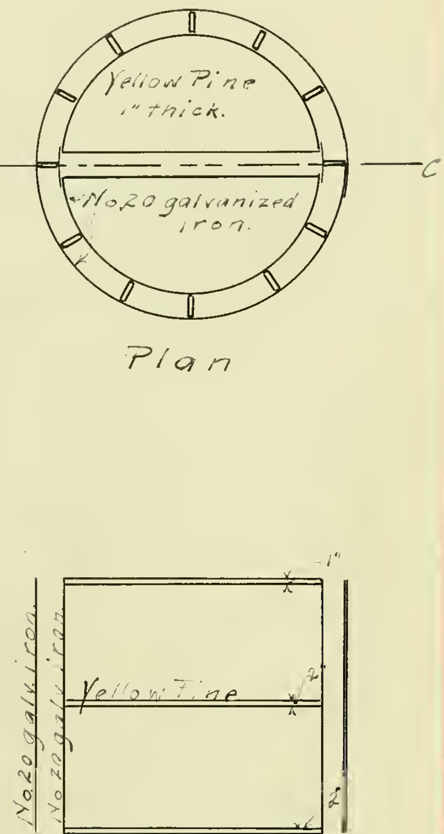
16. CUBES AND BEAMS.

With each set of test pieces made a set of three 6 inch cubes was made from which to obtain the compressive strength of the concrete. Also six beams 8 in. x11 in. x 4 ft., were made, from which to obtain the modulus of rupture.

17. DESCRIPTION AND DISCUSSION OF THE FORMS USED.



Section on A-B
Form for 48" Pipe
Fig. 10.



Section on B-C.
Form for 24" Pipe
Fig. 11.

The forms for these test pieces were of No. 20 galvanized iron nailed upon wooden frames as shown in the accompanying sketch.

These forms answered fairly well the purposes of this investigation but there are several respects in which improvement might be made. It was quite difficult to place and keep the inner form truly circular, and difficult to keep in place the strips of galvanized iron which lapped over the joints where the sections of the form came together. In a continuation of the investigation of sewers it is advised that collapsible steel centers, which are placed on the market and also offered for rent, should be used for this purpose as it is highly probable that they would give a pipe having fewer irregularities and that they would require much less time to put in position. One of the worst defects with the outer form was the means of drawing it up around the inner one. In these experiments No. 12 wire was wrapped around at the top, middle, and bottom respectively and each band twisted to fasten and tighten it. In several cases the wires gave trouble by breaking in the process of twisting while in the case of the two largest pipes the middle wire for each piece broke after the form had been nearly filled, and made considerable trouble. This trouble might be partially remedied by using copper wire which will stand more twisting without breaking, but this would probably

be almost as expensive and certainly not so satisfactory as to use thin iron straps and turn buckles of some sort.

18. BREAKING OF THE TEST PIECES.

In breaking the test sewers the 600,000 lbs. Riehle machine installed in the University of Illinois laboratory of applied mechanics in 1905 and 1906 was used . Since the larger sizes of the pipes weighed about 2200 lbs. and 3800 lbs., respectively, they were lifted from the position in which they were made by means of a rope drawn tightly around the outside and attached to the hook of a differential pulley as shown in the photograph, see frontispiece, placed in a position to roll, and then were rolled into the machine.

As has been before stated it was believed essential in these tests that the manner of loading be accurately known. With this end in view the load was as nearly as possible concentrated upon a longitudinal line along the crown of the pipe and the pipe itself supported upon a longitudinal line on the bottom. To do this a narrow strip (about $1\frac{1}{2}$ inches wide and $\frac{3}{8}$ inch thick) of plaster of Paris was laid along the desired line of support. The test sewer was quickly rolled into position upon it before the plaster of Paris had set. Another strip of plaster of Paris was then laid on the top of the sewer along the line on which the load was to be applied. Upon this, before it had set, was placed a steel bar 3"x$1\frac{1}{2}$"x30" with the 3 inch face in contact with the plaster of Paris. The object of the plaster of Paris both above and below was to give a uniform bearing from end to end of the

test piece. The load was then applied with a downward speed of 1/20 inch per minute until failure occurred.

19. DATA TO BE OBSERVED.

It was desired to observe the extensions of the horizontal diameter and the shortening of the vertical diameter as the load was applied. A micrometer screw reading to one ten thousandth of an inch was used to measure any change in the horizontal diameter. The micrometer was attached as shown in Fig. 12 to a

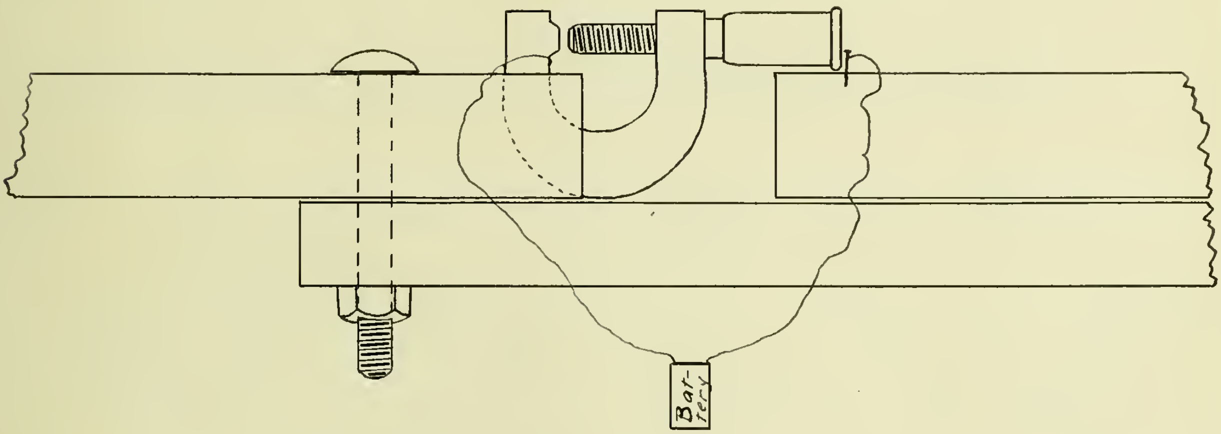


Fig. 12

board secured to one end of the horizontal diameter. A nail was driven into a second board which was secured to the other end of the horizontal diameter. These boards were separated by the lengthening of the horizontal diameter under load and this lengthening measured by the micrometer. By this means the slightest change in the diameter could be detected and measured. An extensometer which read to thousandths of an inch was used to measure the change in vertical diameter. The first test however gave only a slight change (.045 inch) in vertical diameter before

fracture and this probably was due largely to the crushing down of the plaster of Paris in the bearings or else the horizontal diameter would have shown a corresponding increase when as a matter of fact the largest variation observable was .0005 inch. The result of this test was to show the change in these diameters to be so slight that it was not worth while to make these measurements on the remaining tests. In the remaining tests then, the following things were observed. 1. Maximum load. 2. 1st fracture and whether or not it was simultaneous with the maximum load. 3. 2nd fracture. 4. 3rd fracture. 5. 4th fracture.

21. COMPARISON OF RESULTS OF EXPERIMENTS WITH THEORY.

Plate VII shows that the tensile stress is greater at the crown of the sewer than at the end of the horizontal diameter. The tensile stress in the inner fiber at the crown, for this case is about 86% more than that in the outer fiber at the end of the horizontal diameter. This comparison of stresses would lead one to expect the sewer to fail first at the top or at the bottom. The weight of the sewer itself being added to the load and having most of its effect at the bottom makes failure at the botton probable. The points of first failure in the ten tests made were as follows.

Points of 1st failure	No. of failures.
Ends of horizontal diameter	0
Top	2.
Bottom	3.
Simultaneously at top and bottom	4.
Not observed	1.
Total	10.

The fact that all the observed failures occurred at the ends of the vertical rather than the horizontal diameter indicates a conformity of the experiments to theory.

In this connection it is an interesting fact that Prof. Benjamin of Case School of Applied Science made some experiments about 1893 in which he applied a concentrated load to the top of a circular steel hoop until failure occurred and found that in every case failure occurred at the ends of the horizontal diameter. This seems to contradict the theory that has been deduced in this thesis and which so far seems to have been confirmed by the experiments performed in concrete, but the reasons given by Messrs. Filkins and Fort of Cornell University, for this mode of failure are also interesting in this connection. In an article previously referred to (See"Cornell University Association of Civil Engineers,-Transactions" 1895-6 pp 99-112 inc) these gentlemen have deduced a theory of "Stresses and Deflections in Circular Rings under Various Conditions of Loading." The similarity between several of their results and the corresponding results of the theory used in this investigation has been noted on p 20 of this thesis. They likewise come to the same conclusion that the test pieces should fail at the top instead of at the ends of the horizontal diameter. In connection with the failure of the experiments of Prof. Benjamin to confirm their theory they make the following remarks (p. 105 of above named article) "That the maximum stress does occur at A and C (ends of horizontal diameter) seems to be demonstrated experimentally. That theory does not so indicate is probably owing to the neglect of the distortion.

As the load becomes greater, and as the horizontal diameter becomes more and more elongated, the moment M_A becomes greater than that indicated by theory and the stresses due to this moment are correspondingly increased." In addition to this explanation the opinion of Prof. A.N.Talbot of the University of Illinois is given. The above theory being based upon the action of an elastic ring, though it might hold within the elastic limit, probably would not be true beyond that point and these steel rings must have passed the elastic limit before failure occurred, so even though their theory was correct, Prof. Benjamin's experiments would not be well calculated to demonstrate that fact.

If however these are the true causes of the disagreement between theory and experiment, there are two very good reasons for considering concrete an excellent material for testing such a theory; (1) the elongation of the horizontal diameter was so small that it could not be accurately measured, hence the distortion need not be considered and the first error is almost entirely eliminated. (2) While steel reaches its elastic limit long before failure occurs, concrete fails by tension without much stretch and the second source of error is also eliminated. It is therefore to be expected that these pieces would fail as theory indicates if the theory is correct, and such has been shown to be the fact. It seems, then, that we may say that the tests herein described at least point towards the correctness of the theory.

The mere fact however that the test pieces failed in the expected manner is not sufficient to demonstrate the theory. The

loads should agree with the calculated resistance. The compression tests of the cubes show the unit strength to vary from 1210 lbs. per square in., to 2410 lbs. per square in., a variation of 36% from the mean ultimate compressive strength of 1770 lbs. per square in., while the modulus of rupture varies from 310 lbs. per square inch~~es~~ to 400 lbs. per square inch~~es~~ or a variation of 13.3% from the mean of 357 lbs. per square inch~~es~~. With as variable material as this, it is to be expected that a single piece will vary considerably from the calculated strength. The tests in this investigation are only ten in number and only eight of these can be considered as contributing toward the proof or disproof of the theory so far as ultimate load is concerned, the modulus of rupture of the other two being entirely unknown. As the average variation of the maximum loads of these eight pieces was only 18.2% or 5% more than the variation of the modulus of rupture from its mean, it seems reasonable to conclude that in this line also the tests at least point toward the correctness of the theory.

20. DISCUSSION OF RESULTS.

Reinforcing of Concrete Sewers.

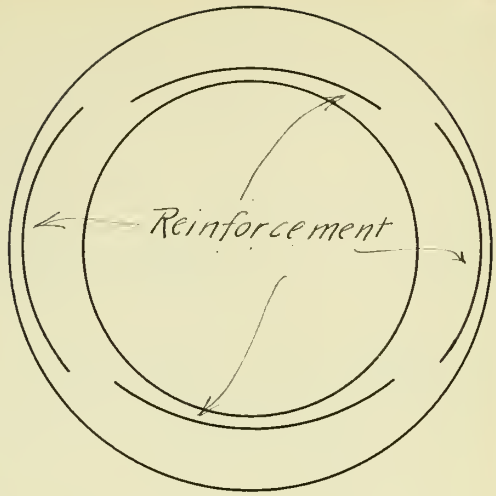


Fig.13

Plate VII shows the places where circumferential reinforcement would be most advantageous to be about as indicated in Fig. 13 because these points receive the maximum tensile stress. As far as the experiments are concerned little can be said as to the effect of reinforcement because only two reinforced concrete sewers were broken and in these the circumferential reinforcement was not placed with a view to increasing the strength but merely to hold in place the longitudinal reinforcement. This longitudinal reinforcement was used to strengthen the joints between sections of the sewer, to enable it to act as a beam if necessary, and to draw the sections closer together. The longitudinal reinforcement was placed at the fifth points of the circle and the circumferential reinforcement at points 7 inches from each end of each section. The most that was learned from this test of reinforcement was that circumferential reinforcing is at least of questionable use on small sewers, as failure probably occurs in tension of the concrete before the steel is stressed and that while failure may occur along the longitudinal reinforcement, this is not a necessary result even though this may be placed at the top or at the bottom. On this account it would be better to avoid having longitudinal reinforcement occur at either top or bottom though this is the place where it would do most effective work if the sewer were called upon to act as a beam.

CHAPTER IV.

22. DESCRIPTIONS OF TABLES AND THEIR USE.

For convenience of reference tables have been compiled from which stresses can be easily determined in any sewer of any size and under any load if loaded either in a longitudinal line along the crown of the sewer or with a uniformly distributed load over the area of the horizontal projection of the sewer.

(a) Table I contains values of H. J. K. M and F for various values of e/r from .001 to .24. As has been pointed out H varies only with e/r and since J K D and F vary only with H, this table can be used for all sizes and loading of sewers. For convenience of reference these values have been plotted in curves of Plates II, III, IV and V respectively.

(b) & (c) Tables II and III. For uniform and concentrated loads of 1440 lbs. per square foot of projected area of the sewer, the values of the stresses in pounds per square inch in the outer and inner fibers at the ends of the horizontal and vertical diameters have been computed, and tabulated in tables II and III. The stresses were taken at these points becuase these are the points where maximum stresses occur, hence failure will occur at these points first and these stresses must therefore govern the design. For convenience these values are plotted in the curves of Plate VI. The following example of the use of this table is given.

Suppose it is desired to know what concentrated load

will cause failure in a section of sewer 3 ft. long having an inside diameter of 60 inches, a thickness of wall of 8 inches and an ultimate tensile strength of 300 lbs. per square inch.

$$e/r = \frac{\frac{1}{2}\text{ thickness of wall}}{\frac{1}{2}\text{ inside } \cancel{\text{radius}} + \frac{1}{2}\text{ thickness of wall}}$$

$$= \frac{\text{thickness of wall}}{\text{inside diameter + thickness of wall}}$$

$$= \frac{8}{60+8} = 8/68 = .118 \text{ (nearly)}$$

Referring to the curve for concentrated load in Plate VI it is seen that (as in all other cases) the maximum tensile stress occurs in the inner fiber at the end of the vertical diameter and in this case is equal to about 740 lbs. per sq. in. for a load of 1440 lbs. per sq. ft. of horizontal projection of sewer. It has been shown that the stresses are proportional to the loads on top. Therefore the load per sq. ft. required to produce a stress of 300 lbs. per sq. in. at the same point is 300/740 x 1440 =584 lbs. per sq. ft. Now the horizontally projected area is equal to the mean diameter multiplied by the length = (60+8) x 36 sq. in.= 17 sq. ft. The load therefore required to cause failure in the sewer is 584x17=9950 lbs.

(d) Table IV. In table IV are tabulated the values of, $\cos\phi$, $\cos\phi - K$, $\cos\phi + D(\cos\phi - K)$ and $\cos\phi + F(\cos\phi - K)$ for e/r=.172 From these values the stresses at any point in the circumference of the sewer can be readily computed for this particular value of e/r.

BREAKING OF CUBES AND BEAMS.

CUBES

(e) Table V is a record of the breaking of the cubes. All the cubes were 6 inches on edges and were broken by compression between two steel plates, a layer of Plaster of Paris having been previously applied both on the top and on the bottom to give an even bearing. The load recorded was the maximum load that could be obtained. In the table below the "Lot" corresponds to the number of the sewer with which it was made.

(f) Table VI BEAMS

The beams broken were 8"x11"x4ft. over all, the span between supports being 42 inches in all cases. In those of lot 5 the load was concentrated at the middle of the span. In the others it was ~~concentrated and~~ applied equally at the one third points (14 inches from each end). All beams were broken with the 11" surface vertical. The weight of each beam effective in breaking may be taken at $\frac{8x11x36}{1728}$ x150=275 lbs.

Moment at center of 275# uniform load = $\frac{wl^2}{8}$

$$= \frac{275x42^2}{8} = \frac{275x42}{8} = 1445.$$

$$S = \frac{Mc}{I}; \quad I=888; \; c=5\tfrac{1}{2}$$

$$c/I \doteq .0062$$

$$S = .0062\ M.$$

In Table VI as in the case of the cubes, the lot number corresponds with the number of the sewer with which the beam was made.

From Tables V and VI is seen that the average compressive strength is about five times the modulus of rupture --about what might have been expected.

<u>Determination of theoretical strength of sewer pieces.</u>

Having determined the average stress at which the concrete may be expected to fail, it is possible to determine the load at which each piece may be expected to fail. Using the modulus of rupture of this concrete as 365lbs per sq. in. a table is made showing the theoretical load required to break the test sewers. The sewers under No. 5 are those made by the Jackson Sewer Pipe Co. of Jackson, Michigan and as its modulus of rupture is not known ~~so~~ the piece may not be expected to break at the theoretical load.

(g) & (h) Tables VII and VIII.

Tables VII and VIII give a comparison of the theoretical and the actual loads to be borne by the sewers broken. Table VII was made from the curves of plate V by the method shown under the explanation of Tables II and III. In table VIII the last number is left out of the average because there was no means of knowing the modulus of rupture for it, that being one of the pieces made by the Jackson sewer pipe Company of Jackson, Michigan. Table VIII shows the average variation of the actual failing load is 19% less than the computed load. It is noticeable that all the pieces with one exception, broke at less than the load computed.

TABLE I

	e/r	H	K	J	D	F
1	.001	.000056				
2	.01	.000006				
3	.03	.00034				
4	.05	.00088	.63673	.2506	55.08	-58.88
5	.06	.00119	.6370	.2508	48.56	-52.64
6	.07	.00170	.63738	.2510	39.43	-43.28
7	..08	.00217	.63758	.2512	35.20	-.39.00
8	.09	.00276	.63796	.2514	30.90	-.34.85
9	.10	.00339	.63835	.2517	27.82	-31.78
10	.12	.00487	.63972	.2519	23.00	-27.00
11	.14	.0066	.64065	.2532	19.7	-23.40
12	.16	.0086	.64173	.2543	16.873	-20.92
13	.18	.0110	.64328	.2552	14.80	-18.70
14	.20	.01369	.64483	.2567	13.17	-17.26
15	.22	.0166	.64631	.2581	11.80	-16.30
16	.24	.0199	.64896	.2598	10.77	-14.86

TABLE II

	e/r	pr/f	Stresses in lbs. per sq. in. $\phi = 0°$ Inner Fiber	$\phi = 0°$ Outer Fiber	$\phi = 90°$ Inner Fiber	$\phi = 90°$ Outer Fiber
1	.05	100.0	1570	1473	1465	1376
2	.06	83.3	1135	1037	1048	958
3	.07	71.4	836	776	774	708
4	.08	62.5	672	602	607	543
5	.09	5[illegible].5	538		481	430
6	.10	50.0	446	398	394	350
7	.12	41.7	313	288	277 ←	248
8	.14	35.7	243	207	211	178
9	.16	31.25	192	161	166	134
10	.18	27.79	[illegible]	124	132	105
11	.20	25.00	130	105	111	85
12	.22	22.72	112	88	96	69
13	.24	20.85	[illegible]	75	81	58

TABLE III.

	e/r	q/2f	Stresses in pounds per sq. in.			
			$\phi = 0°$ Outer fiber.	$\phi = 0°$ Inner fiber.	$\phi = 90°$ Inner fiber.	$\phi = 90°$ Outer fiber.
1.	.05	100.0	2035	2100	3750	3510
2	.06	83.4	1275	1504	2670	2440
3	.07	71.4	1050	1092	1975	1792
4	.08	62.5	822	860	1553	1403
5	.09	55.5	626	678	1235	1096
6	.10	50.0			1010	890
7	.12	41.7	388 ~~[illegible]~~	438 ~~[illegible]~~	720	613
8	.14	35.7	287 ~~[illegible]~~	337 ~~[illegible]~~	535	449
9	.16	31.2	2[illegible]1	265	419	338
10	.18	27.8	174	214	345	265
11	.20	25.0	142	178	278	212
12	.22	22.7	118	154	239	173
13	.24	20.8	100	130	201	145

TABLE IV.

Ref. No.	ϕ in Degrees	Cos ϕ	Cos ϕ -.6427	Cosϕ+ D(cosϕ-k)	Cosϕ+ F(cosϕ-K)
1	0	1.000	0.357	+ 6.6	- 5.95
2	$7\frac{1}{2}$	0.991	0.34[illegible]	+ 6.46	- 5.80
3	15	0.966	0.323	+ 6.05	- 5.33
4	$22\frac{1}{2}$	0.924	0.281	+ 5.33	- 4.56
5	30	0.866	0.223	+ 4.31	- 3.48
6	$37\frac{1}{2}$	0.793	0.1[illegible]0	+ 3.15	- 1.14
7	45	0.707	0.064	+ 1.71	- 0.46
8	$52\frac{1}{2}$	0.609	-.034	+ 1.14	+ 1.27
9	60	0.500	-.143	- 1.75	+ 3.29
10	$67\frac{1}{2}$	0.383	-.260	- 3.70	+ 5.45
11	75	0.259	-.384	- 5.78	+ 7.75
12	$78\frac{3}{4}$	0.195	-.448	- 6.85	+ 8.95
13	$82\frac{1}{2}$	0.131	-.512	- 7.92	+10.13
14	$86\frac{1}{4}$	0.065	-.578	- 9.00	+11.86
15	90	0.000	-.643	-10.10	+12.55

For e/r= .172 D=15.7, F=19.5 & K=.6427

TABLE V.

CRUSHING STRENGTH OF CONCRETE OBTAINED FROM

6" CUBES

Lot	Number	Age When Broken Days	Total Crushing Load Pounds	Pounds per sq. in.
1	1	31	67300	1870
	2	"	62480	1730
	3	"	66700	1850
4	1	28	75700	2100
	2	"	63200	1760
	3	"	79500	2210
5	1	30	53300	1480
	2	"	64100	1780
	3	"	52350	1450
6	1	31	63700	1770
	2	"	56250	1560
	3	"	47150	1310
7	1	30	45500	1260
	2	"	43700	1210
	3	"	60550	1680
8	1	28	79350	2200
	2	"	84050	2330
	3	"	86850	2410
		Average	63500	1770

TABLE VI.

MODULUS OF RUPTURE OBTAINED FROM BEAMS.

Lot	No.	Age when broken --- days	Max. Load -- pounds	Max. moment lb. ft.	Modulus of Rupture lb. per sq. in.
5	1	30	6000	64445	400
	2	"	5850	62945	390
6	1	32	8300	59545	370
	2	"	8200	58845	365
7	1	29	6975	50270	311
	2	"	6950	50095	310
		Average			357.5

TABLE VII.

THEORETICAL BREAKING LOADS.

1	2	3	4	5	6	7	8	9	Average	5
							S = 365			
1	24	3	.111	27×24 649	830	4.33	2810	2600 2460	2530	327
2	24	5 5.5	.172	29×24 696	360	10.0	6960	6870 8100	7435	390
3	48	6	.111	54×24 1298	830	4.33	5630	4610 3820	4210	275
4	48	10	.172	58×24 1392	360	10.0	13920	12850 11320	12085	316
5	15	$1\frac{7}{8}$	.125	25×16½ 422	665	5.41	2280	2955	2955	

Column 1 = Reference No.

" 2 = inside diameter

" 3 = thickness of wall in inches

" 4 = thickness of wall divided by diameter = e/r

" 5 = mean area of horizontal projection of sewer in sq. in.

" 6 = stress in inner fiber at ϕ =90°, load being 10# per sq.in

" 7 = calculated Load in lbs. per sq. in. required to break test piece.

" 8 = calculated Total load on a 24" length required to break test piece

" 9 = actual Load on a 24" length at which test piece broke.

TABLE VIII.

COMPARISON OF THEORETICAL BREAKING LOADS WITH ACTUAL BREAKING LOADS.

Ref. No.	Theoretical Breaking Load	Mean Actual Breaking Load	Difference Pounds	Difference % of Theoretical Breaking Load.
1	2810	2530	280	10.+
2	6960	7435	475	6.8+
3	5630	4215	1415	25.+
4	13920	12085	1835	13.+
5	2280	2955	675	30.-

Mean variation of first four values 18.2+

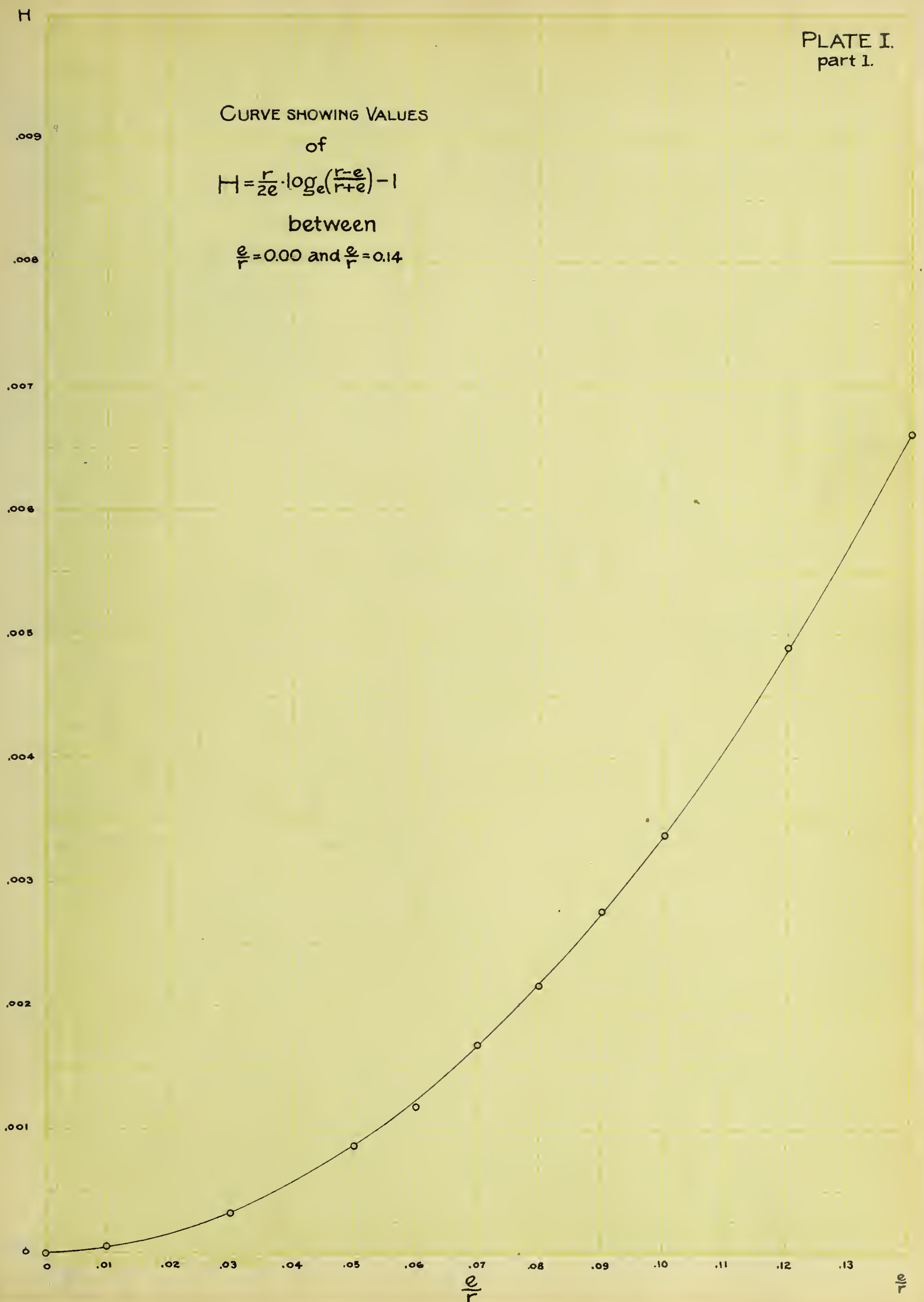


PLATE I.
part 1.
CURVE SHOWING VALUES
of
$H = \frac{r}{2e} \cdot \log_e\left(\frac{r-e}{r+e}\right) - 1$
between
$\frac{e}{r} = 0.00$ and $\frac{e}{r} = 0.14$
H
.009
.008
.007
.006
.005
.004
.003
.002
.001
0
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
$\frac{e}{r}$
$\frac{e}{r}$

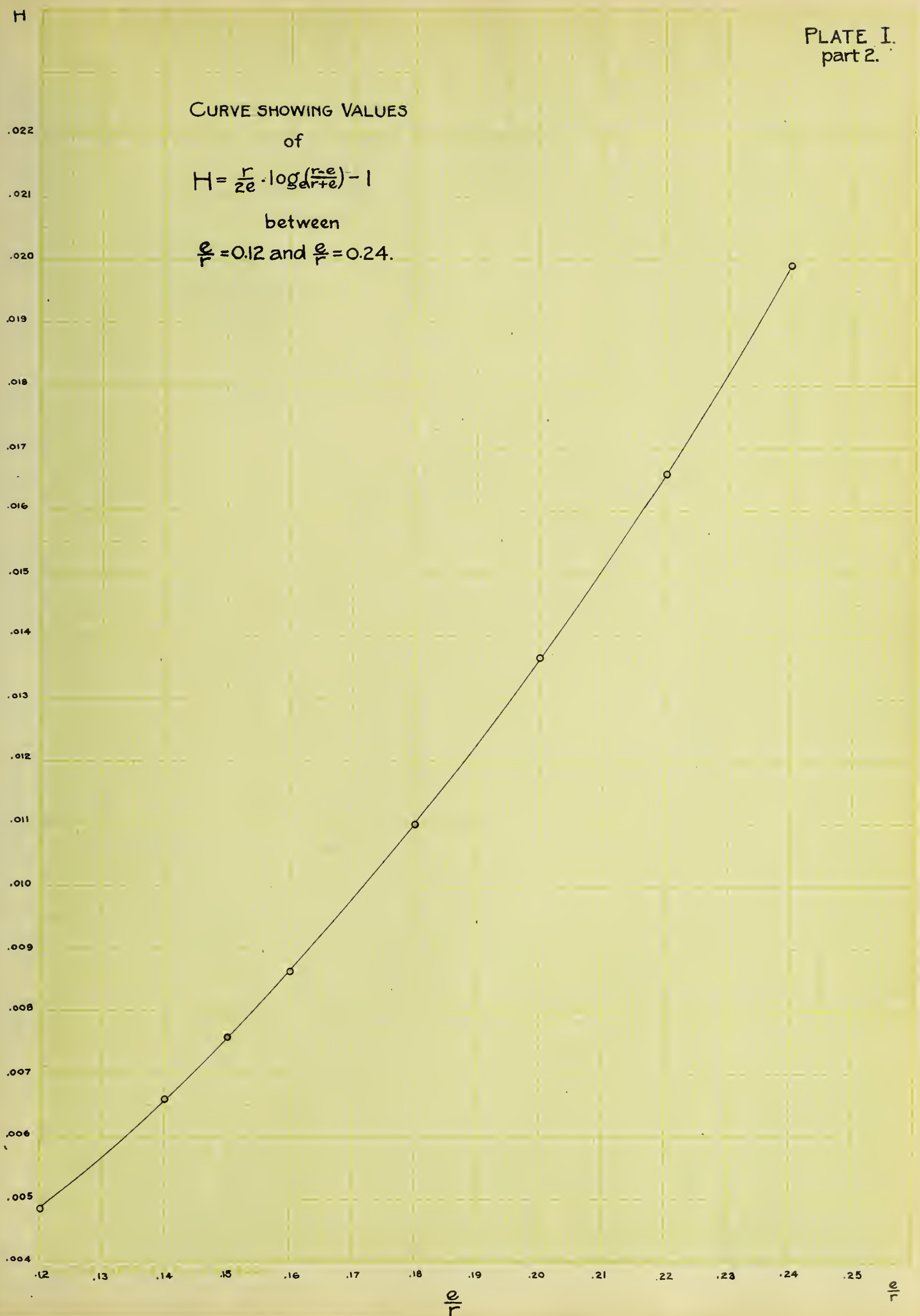


PLATE I.
part 2.
CURVE SHOWING VALUES
of
$H = \frac{r}{2e} \cdot \log_e\left(\frac{r-e}{r+e}\right) - 1$
between
$\frac{e}{r} = 0.12$ and $\frac{e}{r} = 0.24$.
H
.022
.021
.020
.019
.018
.017
.016
.015
.014
.013
.012
.011
.010
.009
.008
.007
.006
.005
.004
.12
.13
.14
.15
.16
.17
.18
.19
.20
.21
.22
.23
.24
.25
$\frac{e}{r}$

PLATE II

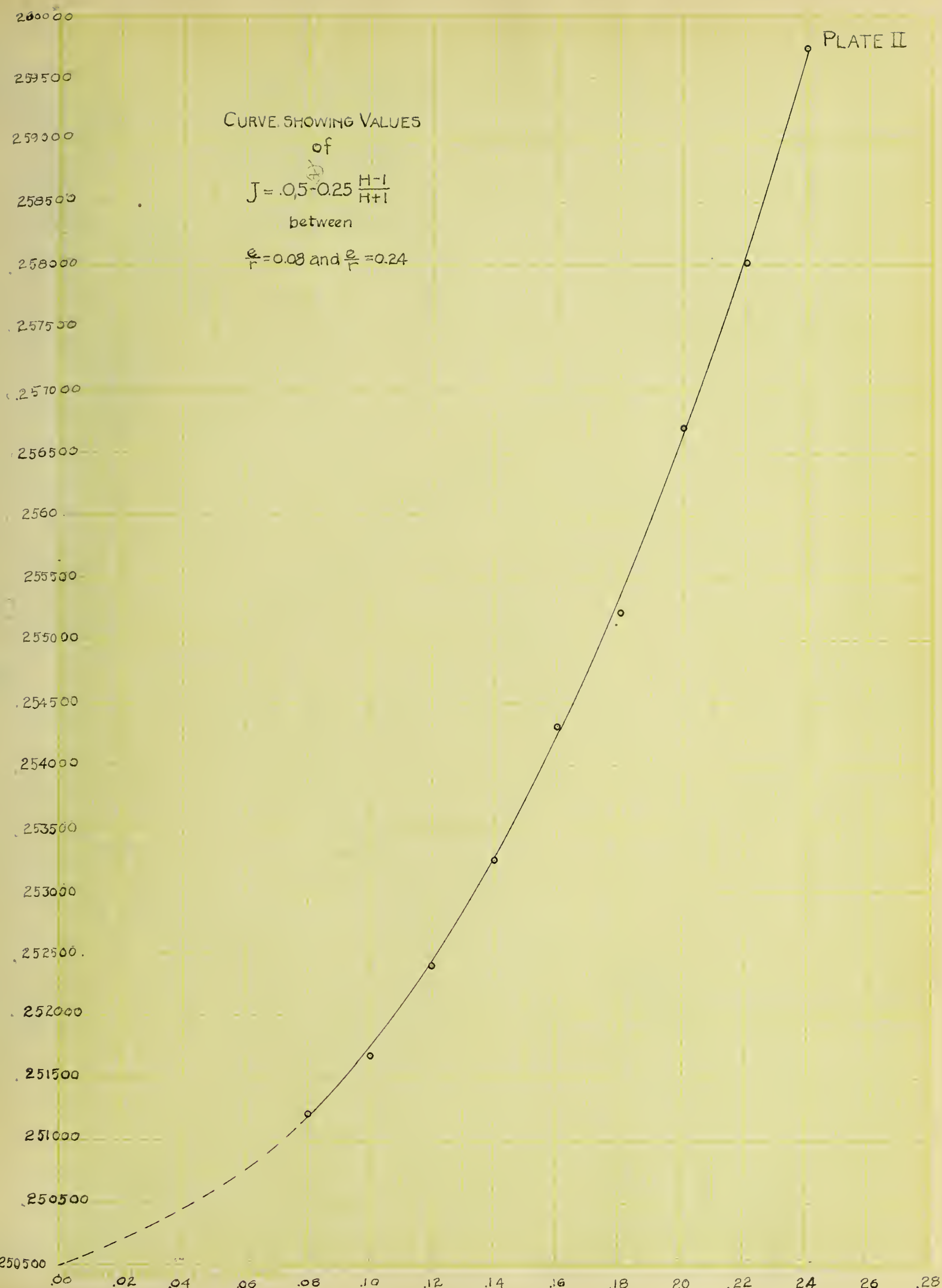

CURVE SHOWING VALUES
of
J = .0,5-0.25 $\frac{H-1}{H+1}$
between
$\frac{e}{r}$ = 0.08 and $\frac{e}{r}$ = 0.24
260000
259500
259000
258500
258000
257500
257000
256500
2560
255500
255000
254500
254000
253500
253000
252500
252000
251500
251000
250500
250500
.00
.02
.04
.06
.08
.10
.12
.14
.16
.18
.20
.22
24
26
.28

PLATE III.

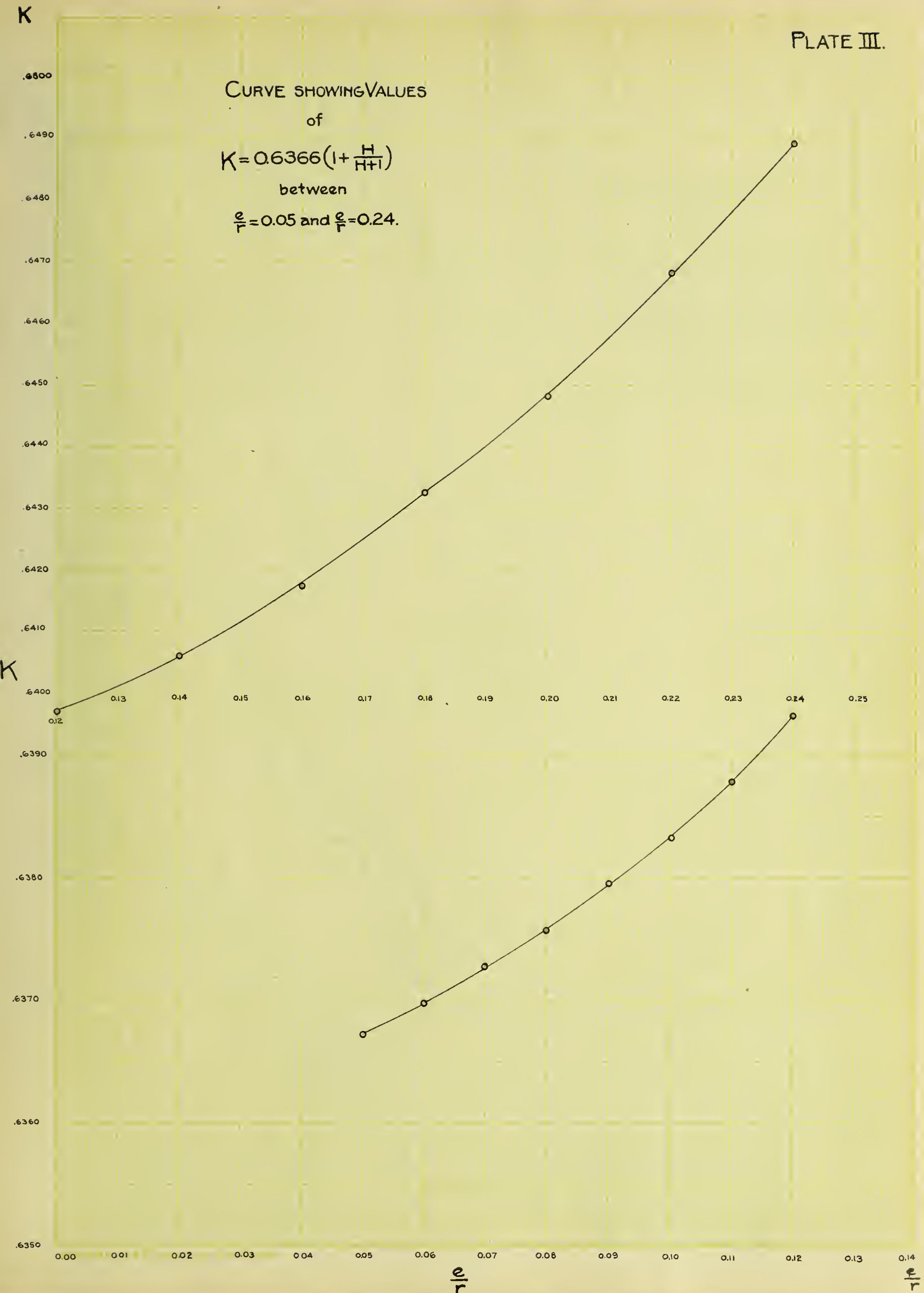

CURVE SHOWING VALUES of $K=0.6366(1+\frac{H}{H+1})$ between $\frac{e}{r}=0.05$ and $\frac{e}{r}=0.24$.
K
.6500
.6490
.6480
.6470
.6460
.6450
.6440
.6430
.6420
.6410
.6400
.6390
.6380
.6370
.6360
.6350
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.20
0.21
0.22
0.23
0.24
0.25
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.11
0.12
0.13
0.14
$\frac{e}{r}$

PLATE IV

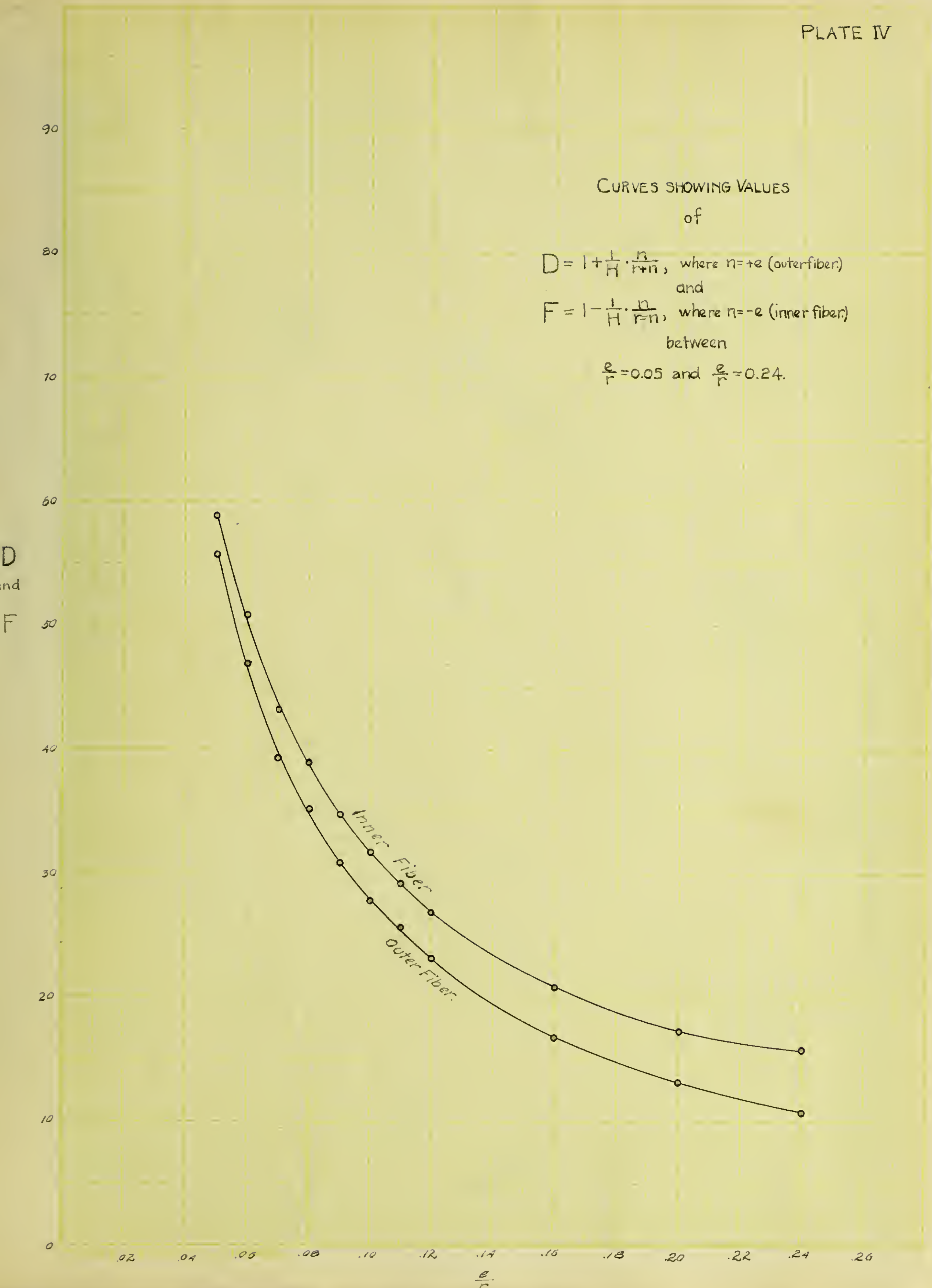

CURVES SHOWING VALUES
of
$D = 1 + \frac{1}{H} \cdot \frac{n}{r+n}$, where $n = +e$ (outerfiber)
and
$F = 1 - \frac{1}{H} \cdot \frac{n}{r-n}$, where $n = -e$ (inner fiber)
between
$\frac{e}{r} = 0.05$ and $\frac{e}{r} = 0.24$.
Inner Fiber
Outer Fiber.
D
and
F
90
80
70
60
50
40
30
20
10
0
.02
.04
.06
.08
.10
.12
.14
.16
.18
.20
.22
.24
.26
$\frac{e}{r}$

Plate V a.

PLATE V,a

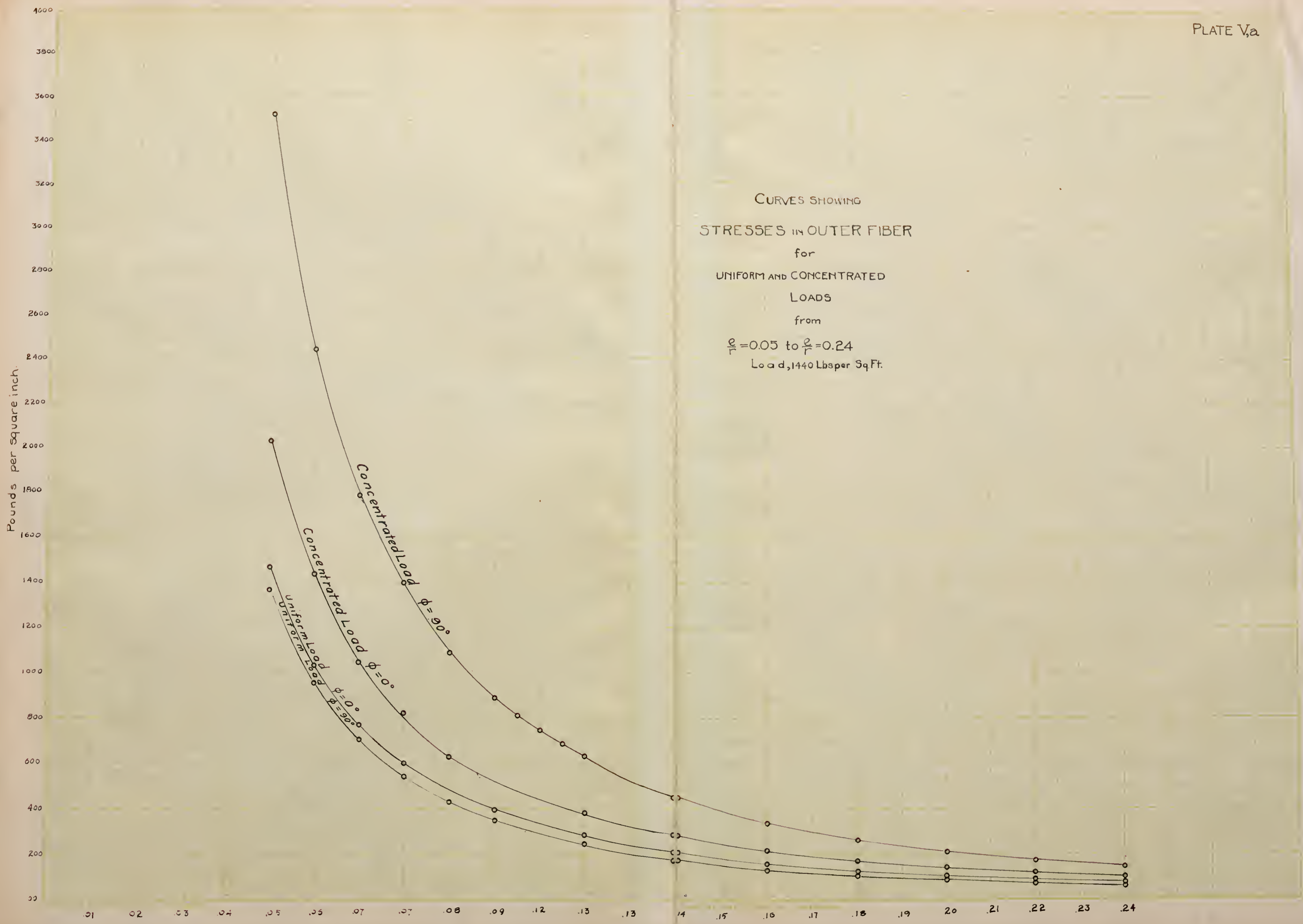

CURVES SHOWING
STRESSES IN OUTER FIBER
for
UNIFORM AND CONCENTRATED
LOADS
from
$\frac{e}{r}=0.05$ to $\frac{e}{r}=0.24$
Load, 1440 Lbs per Sq. Ft.
Pounds per square inch.
Concentrated Load $\phi=90°$
Concentrated Load $\phi=0°$
Uniform Load $\phi=0°$
Uniform Load $\phi=90°$

Plate Vb.

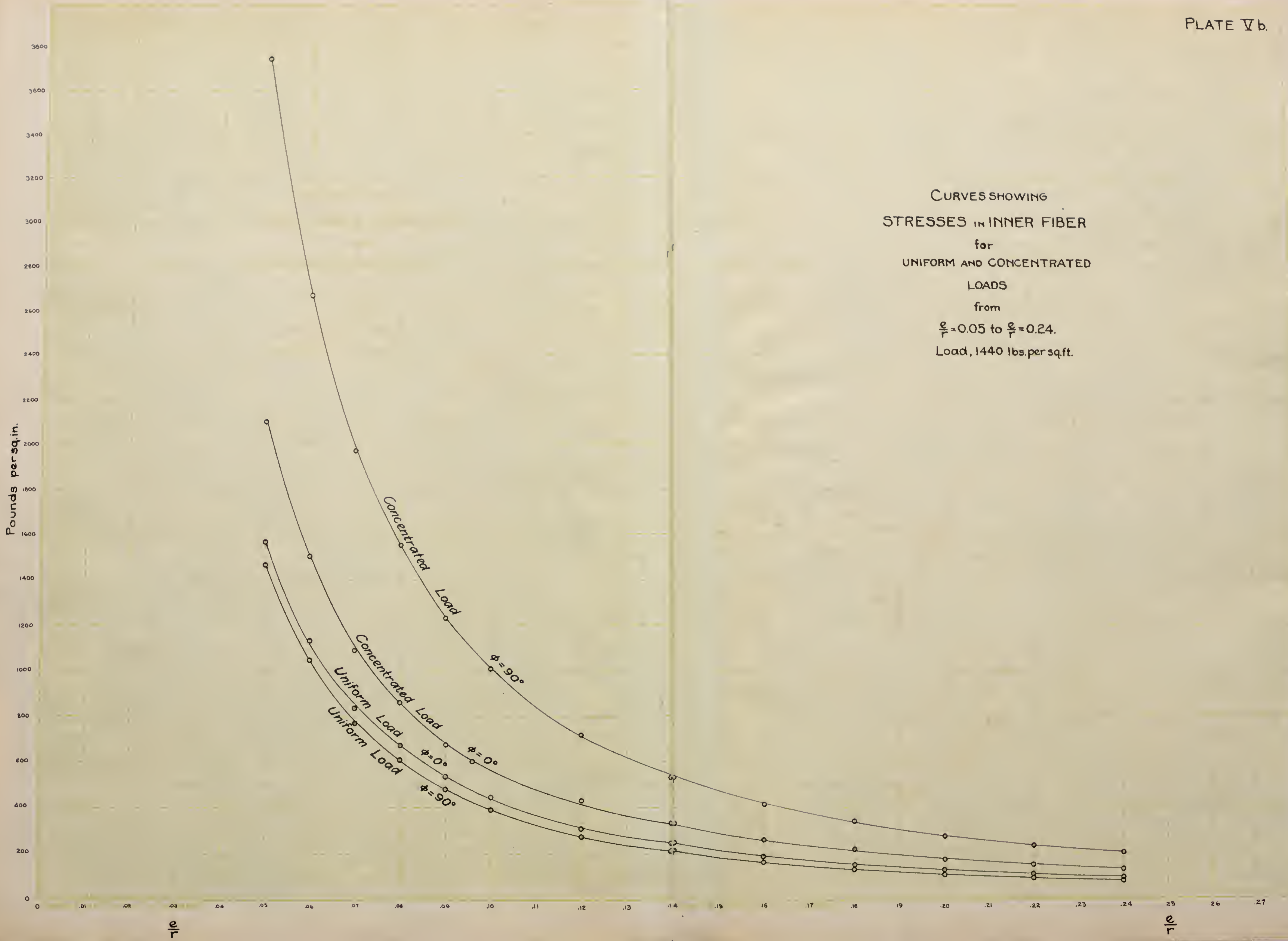
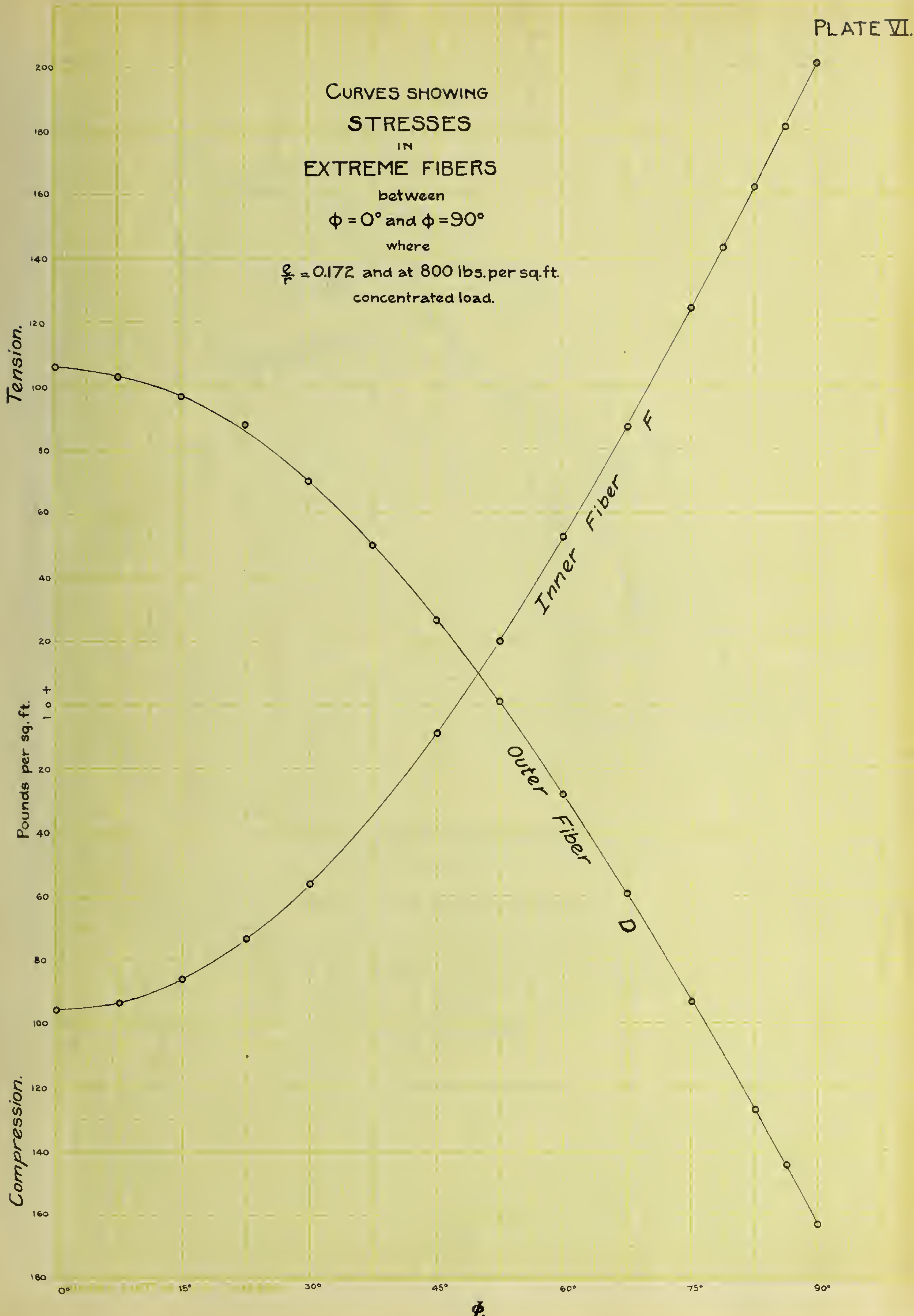
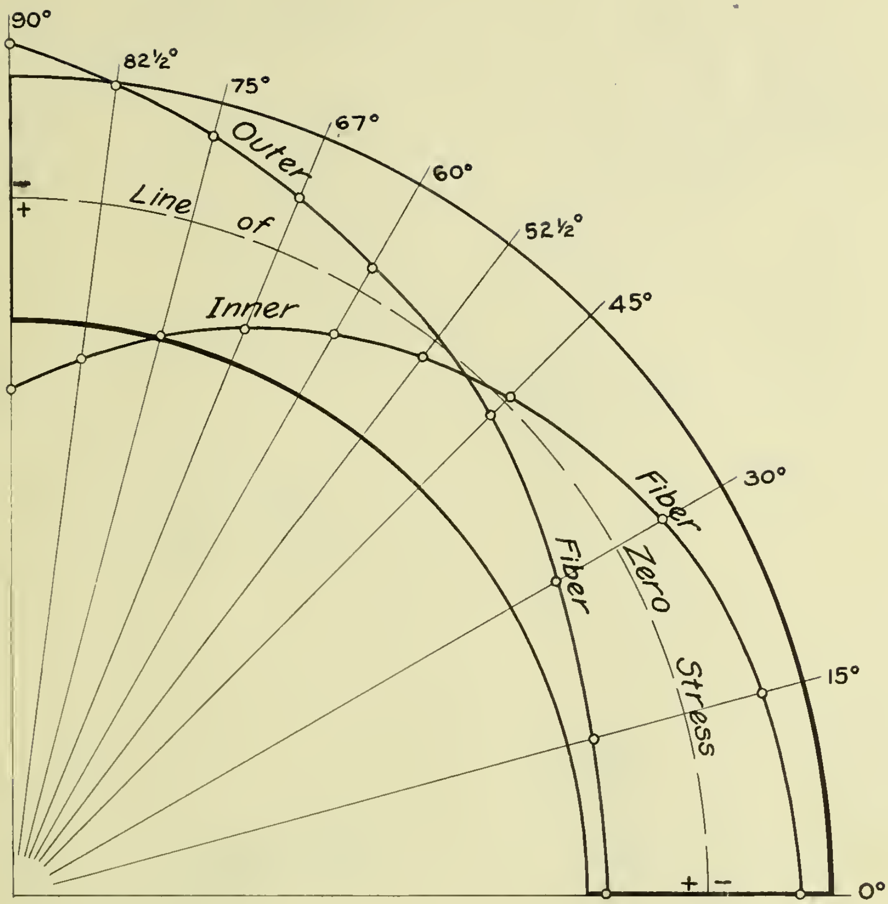

CURVES SHOWING
STRESSES IN INNER FIBER
for
UNIFORM AND CONCENTRATED
LOADS
from
e/r = 0.05 to e/r = 0.24.
Load, 1440 lbs. per sq. ft.
Pounds per sq. in.
Concentrated Load
φ = 90°
Concentrated Load
φ = 0°
Uniform Load
φ = 0°
Uniform Load
φ = 90°
e/r
e/r

PLATE VI.



CURVES SHOWING
STRESSES
IN
EXTREME FIBERS
between
φ = 0° and φ = 90°
where
e/r = 0.172 and at 800 lbs. per sq. ft.
concentrated load.
Tension.
Compression.
Pounds per sq. ft.
Inner Fiber F
Outer Fiber D
200
180
160
140
120
100
80
60
40
20
+ 0 −
20
40
60
80
100
120
140
160
180
0°
15°
30°
45°
60°
75°
90°
Φ.



STRESS DIAGRAM

showing

EXTREME FIBER STRESSES

for

Concentrated Load of 800 lbs. per sq. ft.

between

$\phi = 0°$ and $\phi = 90°$

Scale:– 1 in. = 200 lbs.

PLATE VIII.

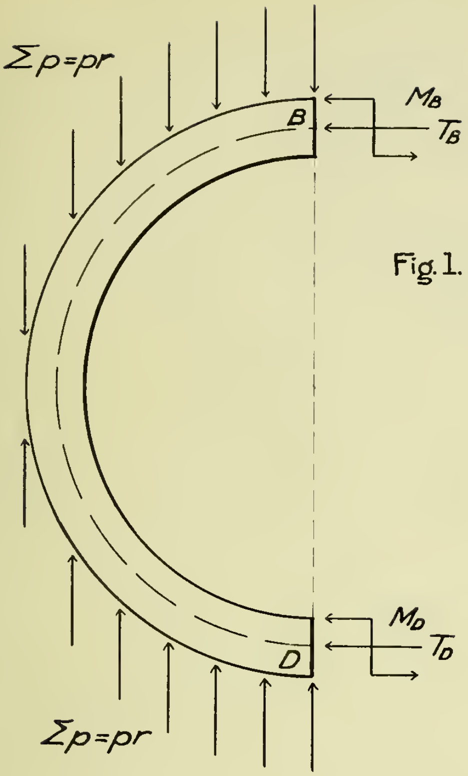


Fig. 1.

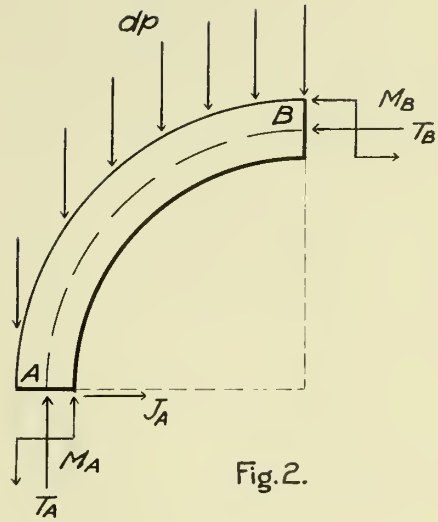


Fig. 2.

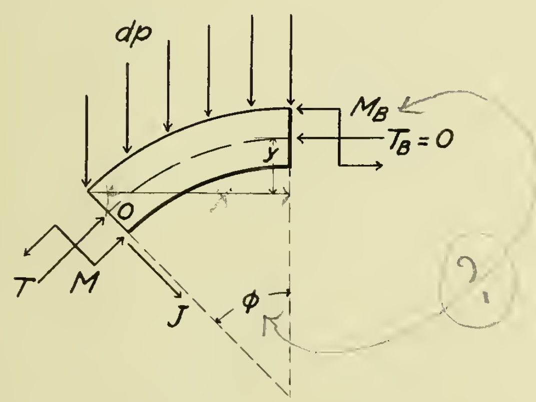


Fig. 4.

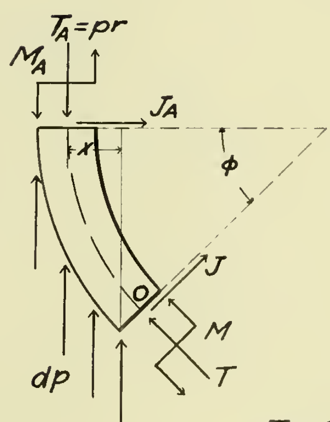


Fig. 3.

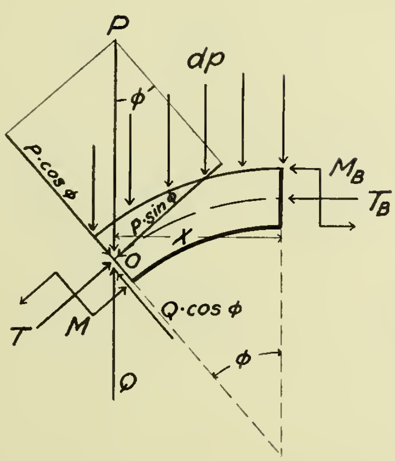


Fig. 5.

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